Baratuci
HW #7

9.116 : Brayton Cycle with Regeneration - 8 pts

A Brayton Cycle with Regeneration using air as the working fluid has a pressure ratio of 7. The minimum and maximum temperatures in the cycle are 310 K and 1150 K. Assuming an isentropic efficiency of 75% for the compressor and 82% for the turbine and an effectiveness of 65% for the regenerator, determine...

a.) The temperature of the turbine effluent.
b.) The net work output, in kJ/kg of air flowing through the system.
c.) The thermal efficiency of the cycle.

Read:
The key to part (a) is to use $\eta_{S,T}$ and the 2nd Gibbs equation to determine $H_5$ and then interpolate on the Ideal Gas Property table for air to determine $T_5$.

The key to part (b) is to use $\eta_{S,C}$ and the 2nd Gibbs equation to determine $H_2$ and then apply the 1st law to the compressor and the turbine to determine the specific shaft work for each. $W_{cycle}$ is the sum of these two shaft work terms.

The key to part (d) is to use the regenerator effectiveness to determine $H_3$ so we can calculate $Q_{34} = Q_H$. Then, we can determine $\eta_{th}$ from its definition using $Q_H$ and $W_{cycle}$ from part (b).

Given:

| $P_2 / P_1$ | 7 |
| $T_4$ | 1150 K |
| $T_1$ | 310 K |
| $\varepsilon_{regen}$ | 65% |

$\eta_{B,comp}$ 75%  
$\eta_{B,turb}$ 82%  
$R$ 8.314 J/mole-K  
MW 28.97 g/mole

Find:

| a.) $T_5$ | ??? K |
| b.) $W_{cycle}$ | ??? kJ/kg |
| c.) $\eta_{th,cycle}$ | ??? |

Diagram:

Assumptions:

1 - The cycle operates at steady-state.
2 - Air is the working fluid and it behaves as an ideal gas.
3 - The Brayton Cycle is modeled as a closed cycle.
4 - The combustor is replaced by a HEX. (External Combustion)
5 - The compressor and turbine are not internally reversible.
6 - Changes in kinetic and potential energies are negligible.
7 - Air has variable specific heats.
8 - The compressor and turbine are adiabatic.
Part a.)

If we can determine the value of $H_5$, we will be able to interpolate on the Ideal Gas Property Table for air and determine $T_5$.

We can determine $H_5$ from the isentropic efficiency for the turbine, as follows.

$$\eta_{S,turb} = \frac{H_4 - H_5}{H_4 - H_{SS}}$$

**Eqn 1**

Solving **Eqn 1** for $H_5$ yields:

$$H_5 = H_4 - \eta_{S,turb} \left( H_4 - H_{SS} \right)$$

**Eqn 2**

Since we know $T_4$, and the enthalpy of an ideal gas depends on $T$ only, we can immediately lookup $H_4$ in the Ideal Gas Property Table for air.

$S_4 = 3.12900 \text{ kJ/kg-K}$  

$H_4 = 1219.25 \text{ kJ/kg}$

Now, we need to determine $H_{SS}$, the enthalpy of the effluent from a hypothetical isentropic turbine. We can do this because we know the values of two intensive variables: $P_5$ and $S_5 = S_4$. The key to using this information is the 2nd Gibbs equation.

$$\hat{S}_{SS} - \hat{S}_4 = \hat{S}_5 - \hat{S}_4 - \frac{R}{MW} \ln \frac{P_5}{P_4} = 0$$

We can then solve **Eqn 3** for the unknown $S_5^{o}$:

$$\hat{S}_5^{o} = \hat{S}_4 + \frac{R}{MW} \ln \frac{P_5}{P_4}$$

**Eqn 4**

Plugging values into **Eqn 4** yields:

$S_5^{o} = 2.57054 \text{ kJ/kg-K}$

Now, we can use $S_5^{o}$ to interpolate on the Ideal Gas Property Table for air to determine $H_{SS}$. We can also determine $T_{SS}$, although it is not necessary.

Next, we can plug values into **Eqn 2**:

Now, we can use $H_5$ to interpolate on the Ideal Gas Property Tables for air to determine $T_5$.

In order to determine the specific shaft work for the cycle, we need to determine the specific shaft work for the compressor and for the turbine.

\[ \hat{W}_{sh,\text{cycle}} = \hat{W}_{sh,\text{comp}} + \hat{W}_{sh,\text{turb}} \] \hspace{1cm} \text{Eqn 5}

We can determine the specific shaft work for the compressor and turbine by applying the 1st Law for open systems to each process.

\[ \dot{Q}_{12} - \hat{W}_{sh,\text{comp}} = \left[ \Delta \hat{H} + \Delta \hat{E}_\text{kin} + \Delta \hat{E}_\text{pot} \right]_{12} \] \hspace{1cm} \text{Eqn 6}

\[ \dot{Q}_{45} - \hat{W}_{sh,\text{turb}} = \left[ \Delta \hat{H} + \Delta \hat{E}_\text{kin} + \Delta \hat{E}_\text{pot} \right]_{45} \] \hspace{1cm} \text{Eqn 7}

Because isentropic efficiencies were given for the compressor and the turbine, it is safe to assume that these devices are adiabatic. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify the 1st Law as applied to the compressor and turbine as follows.

\[ \hat{W}_{sh,\text{comp}} = \hat{H}_1 - \hat{H}_2 \] \hspace{1cm} \text{Eqn 8}

\[ \hat{W}_{sh,\text{turb}} = \hat{H}_4 - \hat{H}_5 \] \hspace{1cm} \text{Eqn 9}

We know the value of \( \hat{H}_4 \) & \( \hat{H}_5 \), so we can plug numbers into Eqn 8 to obtain:

\[ W_{T,\text{isen}} = 507.52 \text{ kJ/kg} \quad W_{T,\text{act}} = 416.17 \text{ kJ/kg} \]

We can lookup \( \hat{H}_1 \), but we must perform an analysis on the compressor that is very similar to the analysis we did on the turbine in part (a).

\( \hat{H}_1 = 310.24 \text{ kJ/kg} \quad S^o_1 = 1.73498 \text{ kJ/kg-K} \)

In order to determine \( \hat{H}_2 \) and \( \hat{H}_4 \), we will use the given isentropic efficiencies of the compressor and turbine.

\[ \eta_{S,\text{comp}} = \frac{-\hat{W}_{sh,\text{isen}}}{-\hat{W}_{sh,\text{act}}} = \frac{\hat{H}_1 - \hat{H}_{2S}}{\hat{H}_1 - \hat{H}_2} \] \hspace{1cm} \text{Eqn 11}

We can solve Eqn 11 for the unknown \( \hat{H}_2 \).

\[ \hat{H}_2 = \hat{H}_1 - \frac{\hat{H}_1 - \hat{H}_{2S}}{\eta_{S,\text{comp}}} \] \hspace{1cm} \text{Eqn 12}

In order to use Eqn 12, we must first determine \( \hat{H}_{2S} \): the enthalpy of the effluent stream from the hypothetical isentropic compressor.

The key to determining \( \hat{H}_{2S} \) and \( \hat{H}_{4S} \) is the application of Gibbs 2nd equation.

\[ \tilde{S}_{2S} - \tilde{S}_1 = \tilde{S}^o_{2S} - \tilde{S}^o_1 - \frac{R}{MW} \ln \frac{P_2}{P_1} = 0 \] \hspace{1cm} \text{Eqn 13}

We can then solve Eqn 13 for the unknowns \( S^o_{2S} \):

\[ \tilde{S}^o_{2S} = \tilde{S}^o_1 + \frac{R}{MW} \ln \frac{P_2}{P_1} \] \hspace{1cm} \text{Eqn 14}

Plugging values into Eqn 14 yields:

\[ S^o_{2S} = 2.29343 \text{ kJ/kg-K} \]
Now, because we know the values of two intensive variables for state 2S (P_2 and S^0_{2S}) and for state 4S (P_4 and S^0_{4S}), we can interpolate on the Ideal Gas Property Table for air to determine H_{2S} and H_{4S}. We can also determine T_{2S} and T_{4S}, although it is not necessary.

<table>
<thead>
<tr>
<th>T (K)</th>
<th>S^0 (kJ/kg-K)</th>
<th>H^0 (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>530</td>
<td>2.27967</td>
<td>533.98</td>
</tr>
<tr>
<td>T_{2S}</td>
<td>2.29343</td>
<td>H_{2S}</td>
</tr>
<tr>
<td>540</td>
<td>2.29906</td>
<td>544.35</td>
</tr>
</tbody>
</table>

Next, we can plug values into Eqn 12:

Now, we can plug values into Eqns 8 & 5, in that order.

\[ W_{c,isen} = -231.10 \text{ kJ/kg} \]
\[ W_{c,act} = -308.13 \text{ kJ/kg} \]
\[ W_{cycle} = 108.04 \text{ kJ/kg} \]

**Part c.**

The thermal efficiency of the cycle is defined by:

\[ \eta_{th} = \frac{\dot{W}_{cycle}}{\dot{Q}_H} \quad \text{Eqn 15} \]

We found \( \dot{W}_{cycle} \) in part (b), so all we need to do here is determine \( \dot{Q}_H = Q_{23} \). We can accomplish this by applying the steady-state form of the 1st Law for open systems to the combustor, as follows.

\[ \dot{Q}_{34} - \dot{W}_{sh,34} = [\Delta H + \Delta E_{kin} + \Delta E_{pot}]_{34} \quad \text{Eqn 16} \]

The combustor is a HEX with no moving parts, so it is safe to assume that no shaft work occurs in the combustor. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify the 1st Law as applied to the combustor as follows.

\[ \dot{Q}_{34} = \dot{H}_4 - \dot{H}_3 \quad \text{Eqn 17} \]

We need to determine \( H_3 \) before we can use Eqns 17 & 15 to complete this problem.

We can determine \( H_3 \) using the regenerator effectiveness and its definition.

\[ \epsilon_{regen} = \frac{\dot{H}_1 - \dot{H}_2}{\dot{H}_3 - \dot{H}_2} \quad \text{Eqn 18} \]

Solving Eqn 18 for the unknown \( H_3 \) yields:

\[ \dot{H}_3 = \dot{H}_2 + \epsilon_{regen} (\dot{H}_3 - \dot{H}_2) \quad \text{Eqn 19} \]

Plugging values into Eqns 19, 17 & 15, in that order, yields:

\[ H_3 = 738.43 \text{ kJ/kg} \]
\[ Q_{34} = Q_{4i} = 480.81 \text{ kJ/kg} \]

**Verify:**
None of the assumptions made in this problem solution can be verified.

**Answers:**

a.) \[ T_5 = 783 \text{ K} \]
b.) \[ W_{cycle} = 108.04 \text{ kJ/kg} \]
c.) \[ \eta_{th, cycle} = 22.5\% \]

**Note:**
We could also calculate \( T \) and \( S^0 \) for stream 2, just to be thorough.
A gas refrigeration cycle with a pressure ratio of 3 uses helium as the working fluid. The temperature of the helium is -10°C at the compressor inlet and 50°C at the turbine inlet. Assuming adiabatic efficiencies of 80% for both the turbine and the compressor, determine...

a.) The **minimum temperature** in the cycle.
b.) The **coefficient of performance**.
c.) The **mass flow rate of the helium** in kg/s for a refrigeration load of 18 kW.

### Data

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p )</td>
<td>5.1926 kJ/kg-K</td>
</tr>
<tr>
<td>( C_v )</td>
<td>3.1156 kJ/kg-K</td>
</tr>
</tbody>
</table>

### Read

The keys to this problem are the isentropic efficiencies, the assumption that helium behaves as an ideal gas and the fact that the heat capacities, and therefore the heat capacity ratio, are constant.

The key to determining the mass flow rate of helium in part (c) is the given value for the refrigeration load.

### Given

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_2 )</td>
<td>-10 °C</td>
</tr>
<tr>
<td>( T_4 )</td>
<td>50 °C</td>
</tr>
<tr>
<td>( T_{263} )</td>
<td>263.15 K</td>
</tr>
<tr>
<td>( T_{323} )</td>
<td>323.15 K</td>
</tr>
<tr>
<td>( \eta_{\text{comp}} )</td>
<td>80%</td>
</tr>
<tr>
<td>( \eta_{\text{turb}} )</td>
<td>80%</td>
</tr>
<tr>
<td>( Q_C )</td>
<td>18.0 kW</td>
</tr>
</tbody>
</table>

### Find

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 )</td>
<td>??? °C</td>
</tr>
<tr>
<td>( \text{COP}_R )</td>
<td>???</td>
</tr>
<tr>
<td>( m )</td>
<td>??? kg/s</td>
</tr>
</tbody>
</table>

### Diagram

![Diagram of Helium Gas Refrigeration Cycle](image)

### Assumptions

1. The cycle operates at steady-state.
2. Helium is the working fluid and it behaves as an ideal gas.
3. The Brayton Refrigeration Cycle is modeled as as a closed cycle.
4. The combustor is replaced by a HEX. (External Combustion)
5. The compressor and turbine are not internally reversible.
6. Changes in kinetic and potential energies are negligible.
7. Helium has constant specific heats.
8. The compressor and turbine are adiabatic.

### Equations / Data / Solve

**Part a.)** The key to determining \( T_1 \) is the isentropic efficiency of the turbine.

\[
\eta_{\text{S,turb}} = 1 - \frac{-\dot{W}_{\text{Sh,act}}}{-\dot{W}_{\text{Sh,isen}}} = \frac{\dot{H}_4 - \dot{H}_1}{\dot{H}_4 - \dot{H}_{1S}} \quad \text{Eqn 1}
\]

Because the heat capacities are constant, the change in enthalpy in the numerator and denominator can be expressed as follows:

\[
\eta_{\text{S,turb}} = \frac{\dot{H}_4 - \dot{H}_1}{\dot{H}_4 - \dot{H}_{1S}} = \frac{C_P (T_4 - T_1)}{C_P (T_4 - T_{1S})} = \frac{T_4 - T_1}{T_4 - T_{1S}} \quad \text{Eqn 2}
\]

The heat capacity cancels in Eqn 2 and we can solve for the unknown \( T_1 \):

\[
T_1 = T_4 - \eta_{\text{S,turb}} (T_4 - T_{1S}) \quad \text{Eqn 3}
\]
We still need to determine \( T_{1S} \) (the temperature of the effluent from a hypothetical isentropic turbine) before we can use Eqn 3. For an isentropic process in which the fluid is an ideal gas with constant heat capacities, the following PVT relationship applies.

\[
T_{1S} \frac{l-\gamma}{\gamma} = T_4 \frac{l-\gamma}{\gamma} = \text{constant}
\]

Eqn 4

Where:

\[
\gamma = \frac{\hat{c}_p}{\hat{c}_v}
\]

\[
\gamma = 1.667
\]

We can solve Eqn 4 for \( T_{1S} \):

\[
T_{1S} = T_4 \left( \frac{p_4}{p_1} \right)^{\frac{l-\gamma}{\gamma}}
\]

Eqn 5

Plugging values into Eqns 5 & 3 yields:

\[
\begin{align*}
\frac{\gamma - 1}{\gamma} &\approx 0.4000 \\
T_1 &\approx 231.2 \text{ K}
\end{align*}
\]

Part b.)

We can determine the \( \text{COP}_{R} \) from its definition.

\[
\text{COP}_{R} = \frac{\dot{Q}_C}{-\dot{W}_{\text{cycle}}}
\]

Eqn 6

The value of \( \dot{Q}_C \) is given, so we need to determine \( \dot{W}_{\text{cycle}} \). Shaft work only occurs at the compressor and the turbine.

\[
\dot{W}_{\text{cycle}} = \dot{W}_{\text{sh, comp}} + \dot{W}_{\text{sh, turb}}
\]

Eqn 7

We can determine the specific shaft work for the compressor and turbine by applying the steady-state form of the 1st Law for open systems to each process.

\[
\dot{Q}_{23} - \dot{W}_{\text{sh, comp}} = \left[ \Delta \hat{H} + \Delta \hat{E}_{\text{kin}} + \Delta \hat{E}_{\text{pot}} \right]_{23}
\]

Eqn 8

\[
\dot{Q}_{41} - \dot{W}_{\text{sh, turb}} = \left[ \Delta \hat{H} + \Delta \hat{E}_{\text{kin}} + \Delta \hat{E}_{\text{pot}} \right]_{41}
\]

Eqn 9

Because isentropic efficiencies were given for the compressor and the turbine, it is safe to assume that these devices are adiabatic. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify the 1st Law as applied to the compressor and turbine as follows.

\[
\dot{W}_{\text{sh, comp}} = \dot{H}_2 - \dot{H}_3
\]

Eqn 10

\[
\dot{W}_{\text{sh, turb}} = \dot{H}_4 - \dot{H}_1
\]

Eqn 11

Because the heat capacities are constant, the change in enthalpy in Eqns 10 & 11 can be expressed as follows:

\[
\dot{W}_{\text{sh, comp}} = \hat{c}_p (T_2 - T_3)
\]

Eqn 12

\[
\dot{W}_{\text{sh, turb}} = \hat{c}_p (T_4 - T_1)
\]

Eqn 13

We still need to determine \( T_3 \) in order to use Eqn 12. The procedure is analogous to the one used to determine \( T_1 \) in part (a). The key to determining \( T_3 \) is the isentropic efficiency of the turbine.

\[
\eta_{\text{s, comp}} = \frac{-\dot{W}_{\text{sh, isen}}}{-\dot{W}_{\text{sh, act}}} = \frac{\dot{H}_2 - \dot{H}_{3S}}{\dot{H}_2 - \dot{H}_3}
\]

Eqn 14

Because the heat capacities are constant, the change in enthalpy in the numerator and denominator can be expressed as follows:

\[
\eta_{\text{s, comp}} = \frac{\dot{H}_2 - \dot{H}_{3S}}{\dot{H}_2 - \dot{H}_3} = \frac{\hat{c}_p (T_2 - T_{3S})}{\hat{c}_p (T_2 - T_3)} = \frac{T_2 - T_{3S}}{T_2 - T_3}
\]

Eqn 15
We can solve **Eqn 17** for $T_{3s}$:

$$T_{3s} = T_2 \left( \frac{P_2}{P_3} \right)^{\frac{1 - \gamma}{\gamma}}$$  \hspace{1cm} \textbf{Eqn 18}

We can plug values into **Eqns 18 & 16** yields:

<table>
<thead>
<tr>
<th>$T_{3s}$</th>
<th>$408.4$ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_3$</td>
<td>$444.67$ K</td>
</tr>
</tbody>
</table>

Now, we can plug values into **Eqns 12 & 13** to obtain:

- $W_{\text{comp}}$ = $-942.6$ kJ/kg
- $W_{\text{turb}}$ = $477.4$ kJ/kg

We still need to determine the specific heat transfer at HEX #1, $Q_C$, so that we can use **Eqn 6** to complete part (b). We can determine $Q_C$ by applying the steady-state form of the 1st Law for open systems to HEX #1.

$$\dot{Q}_{12} = \dot{W}_{\text{sh}} = \left[ \Delta \dot{H} + \Delta \dot{E}_{\text{kin}} + \Delta \dot{E}_{\text{pot}} \right]_{12}$$  \hspace{1cm} \textbf{Eqn 19}

The shaft work for a HEX is zero because there are no moving parts. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify the 1st Law as applied to HEX #1 as follows.

$$\dot{Q}_{12} = \dot{H}_2 - \dot{H}_1$$  \hspace{1cm} \textbf{Eqn 20}

Because the heat capacities are constant, the change in enthalpy in **Eqn 20** can be expressed as follows:

$$\dot{Q}_{12} = \dot{H}_2 - \dot{H}_1 = \dot{C}_p (T_2 - T_1)$$  \hspace{1cm} \textbf{Eqn 21}

Now, we can plug values into **Eqns 21, 7 & 6**:

- $Q_C = 165.8$ kJ/kg
- $W_{\text{cycle}}$ = $-465.2$ kJ/kg

**Part c.**

We can determine the mass flow rate of helium through the cycle because we know both the heat transfer rate and the specific heat transfer at HEX #1. The key relationship between these variables is:

$$\dot{m} = \frac{\dot{Q}}{\dot{Q}_C}$$  \hspace{1cm} \textbf{Eqn 22}

Plugging values into **Eqn 22** yields:

$$\dot{m} = 0.1086 \text{ kg/s}$$

None of the assumptions made in this problem solution can be verified.

**Answers:**

- a.) $T_1$ = $-41.9$ °C
- b.) $\text{COP}_R$ = 35.6%
- c.) $\dot{m}$ = 0.109 kg/s
A gas turbine power plant operates on the basic Brayton Cycle. Air is the working fluid and the cycle delivers 15 MW of power. The minimum and maximum temperatures in the cycle are 310 K and 900 K, respectively. The air pressure at the compressor outlet is 8 times the pressure at the compressor inlet. Assuming an isentropic efficiency of 80% for the compressor and 86% for the turbine, determine the mass flow rate of air through the cycle. Assume that air behaves as an ideal gas, but do not assume that the heat capacities of the air are constants.

Read:

The key to determining the mass flow rate of air through the cycle is the given value of the net power output for the cycle.

We need to determine the specific shaft work for the pump and the turbine. The net specific shaft work for the cycle is the sum of these two. The mass flow rate is the ratio of the power output to the net specific work. Check the units!

The values of \( H_2 \) and \( H_4 \) can be determined from \( H_{2S} \) and \( H_{4S} \) using the isentropic efficiency.

**Given:**

- \( W_{cycle} = 15000 \text{ kW} \)
- \( T_2 = 900 \text{ K} \)
- \( T_4 = 310 \text{ K} \)
- \( P_2/P_4 = 8 \)

**Find:**

- \( m \) \( ??? \) \( \text{kg/s} \)

**Diagram:**

![Diagram of a Brayton Cycle](image)

**Assumptions:**

1. The cycle operates at steady-state.
2. Air is the working fluid and it behaves as an ideal gas.
3. The Brayton Cycle is modeled as a closed cycle.
4. The combustor is replaced by a HEX. (External Combustion)
5. The compressor and turbine are not internally reversible.
6. Changes in kinetic and potential energies are negligible.
7. Air has variable specific heats.
8. The compressor and turbine are adiabatic.

**Equations / Data / Solve:**

I like to organize the properties of the streams in more complex problems, like this one, in a table. This helps me keep track of what I know and what I do not know as I progress through the solution. The cells in the table below are color-coded.

<table>
<thead>
<tr>
<th>Stream #</th>
<th>( T ) (K)</th>
<th>( H^0 ) (kJ/kg)</th>
<th>( S^0 ) (kJ/kg-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1S</td>
<td>557.3</td>
<td>562.61</td>
<td>2.33175</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>625.70</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>900</td>
<td>932.93</td>
<td>2.8486</td>
</tr>
<tr>
<td>3S</td>
<td>515.9</td>
<td>519.37</td>
<td>2.25179</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>577.27</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>310</td>
<td>310.24</td>
<td>1.73498</td>
</tr>
</tbody>
</table>

**Color Codes:**

- **Given**
- **Lookup**
- **Key Calcs**
- **Key Calcs**
- **Interpolate**
- **Interpolate**
The key to determining the mass flow rate of air through the cycle is the given value for the net work of the cycle: 15 MW. This value is related to the mass flow rate of air by the following equation.

\[
\dot{W}_{\text{cycle}} = \dot{m} \left( \dot{W}_{\text{comp}} + \dot{W}_{\text{turb}} \right) \tag{Eqn 1}
\]

Solving for the mass flow rate yields:

\[
\dot{m} = \frac{\dot{W}_{\text{cycle}}}{\dot{W}_{\text{comp}} + \dot{W}_{\text{turb}}} \tag{Eqn 2}
\]

We can determine the specific work for the compressor and the turbine by applying the 1st Law to each.

\[
\dot{Q}_{41} - \dot{W}_{\text{sh,comp}} = \left[ \Delta \hat{H} + \Delta \hat{E}_\text{kin} + \Delta \hat{E}_\text{pot} \right]_{41} \tag{Eqn 3}
\]

\[
\dot{Q}_{23} - \dot{W}_{\text{sh,turb}} = \left[ \Delta \hat{H} + \Delta \hat{E}_\text{kin} + \Delta \hat{E}_\text{pot} \right]_{23} \tag{Eqn 4}
\]

Because isentropic efficiencies were given for the compressor and the turbine, it is safe to assume that these devices are adiabatic. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify the 1st Law as applied to the compressor and turbine as follows.

\[
\dot{W}_{\text{sh,comp}} = \hat{H}_4 - \hat{H}_1 \tag{Eqn 5}
\]

\[
\dot{W}_{\text{sh,turb}} = \hat{H}_2 - \hat{H}_3 \tag{Eqn 6}
\]

Now, we need to determine the enthalpy of each of the four streams that make up this cycle.

We can immediately do this for streams 2 & 4 because we know their temperature and we have assumed that they are ideal gases. Enthalpy of ideal gases is a function of temperature only, so we can lookup \(H_2\) and \(H_4\) in the Ideal Gas Property Table for air. We might as well lookup \(S^0_2\) and \(S^0_4\) at the same time.

<table>
<thead>
<tr>
<th>Stream</th>
<th>Enthalpy (kJ/kg)</th>
<th>Entropy (kJ/kg-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_2)</td>
<td>310.24</td>
<td>(S^0_2) 1.73498</td>
</tr>
<tr>
<td>(H_4)</td>
<td>932.93</td>
<td>(S^0_4) 2.84856</td>
</tr>
</tbody>
</table>

In order to determine \(H_1\) and \(H_3\), we will use the given isentropic efficiencies of the compressor and turbine.

\[
\eta_{S,\text{comp}} = \frac{-\dot{W}_{\text{sh,isen}}}{-\dot{W}_{\text{sh,act}}} = \frac{\hat{H}_4 - \hat{H}_{1S}}{\hat{H}_4 - \hat{H}_1} \tag{Eqn 7}
\]

\[
\eta_{S,\text{turb}} = \frac{-\dot{W}_{\text{sh,act}}}{-\dot{W}_{\text{sh,isen}}} = \frac{\hat{H}_2 - \hat{H}_3}{\hat{H}_2 - \hat{H}_{3S}} \tag{Eqn 8}
\]

We can solve **Eqns 7 & 8** for the unknowns \(H_1\) and \(H_3\).

\[
\hat{H}_1 = \hat{H}_4 - \frac{\hat{H}_4 - \hat{H}_{1S}}{\eta_{S,\text{comp}}} \tag{Eqn 9}
\]

\[
\hat{H}_3 = \hat{H}_2 - \eta_{S,\text{turb}} \left( \hat{H}_2 - \hat{H}_{3S} \right) \tag{Eqn 10}
\]
In order to use Eqns 9 & 10, we must first determine \( H_{1S} \) and \( H_{3S} \), the enthalpy of the effluent streams from the hypothetical isentropic compressor and turbine.

The key to determining \( H_{1S} \) and \( H_{3S} \) is the application of Gibbs 2nd equation.

\[
\hat{S}_{1S} - \hat{S}_4 = \hat{S}_{1S}^o - \hat{S}_4^o - \frac{R}{MW} \ln \frac{P_1}{P_4} = 0 \quad \text{Eqn 11}
\]

\[
\hat{S}_{3S} - \hat{S}_2 = \hat{S}_{3S}^o - \hat{S}_2^o - \frac{R}{MW} \ln \frac{P_3}{P_2} = 0 \quad \text{Eqn 12}
\]

We can then solve Eqns 11 & 12 for the unknowns \( S_{1S}^o \) and \( S_{3S}^o \):

\[
\hat{S}_{1S}^o = \hat{S}_4^o + \frac{R}{MW} \ln \frac{P_1}{P_4} \quad \text{Eqn 13}
\]

\[
\hat{S}_{3S}^o = \hat{S}_2^o + \frac{R}{MW} \ln \frac{P_3}{P_2} \quad \text{Eqn 14}
\]

Plugging values into Eqns 13 & 14 yields:

\[
S_{1S}^o \quad 2.33175 \text{ kJ/kg-K}
\]

\[
S_{3S}^o \quad 2.25179 \text{ kJ/kg-K}
\]

Now, because we know the value \( S_{1S}^o \) and \( S_{3S}^o \), we can interpolate on the Ideal Gas Property Table for air to determine \( H_{1S} \) and \( H_{3S} \). We can also determine \( T_{1S} \) and \( T_{3S} \), although it is not necessary.

<table>
<thead>
<tr>
<th>T</th>
<th>S^o</th>
<th>H^o</th>
</tr>
</thead>
<tbody>
<tr>
<td>550</td>
<td>2.31809</td>
<td>555.74</td>
</tr>
<tr>
<td>560</td>
<td>2.33685</td>
<td>565.17</td>
</tr>
</tbody>
</table>

\[
T_{1S} \quad 557.3 \text{ K}
\]

\[
T_{3S} \quad 515.9 \text{ K}
\]

<table>
<thead>
<tr>
<th>H_{1S}</th>
<th>562.61 \text{ kJ/kg}</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_{3S}</td>
<td>519.37 \text{ kJ/kg}</td>
</tr>
</tbody>
</table>

Now, we know all the values we need to make use of Eqns 7 - 10.

\[
W_{c,\text{sten}} \quad -252.37 \text{ kJ/kg}
\]

\[
W_{c,\text{act}} \quad -315.46 \text{ kJ/kg}
\]

\[
H_1 \quad 625.70 \text{ kJ/kg}
\]

Finally, we can plug values back into Eqn 2 to determine the mass flow rate of air through the cycle.

\[
W_{\text{cycle}} \quad 40.20 \text{ kJ/kg}
\]

\[
m \quad 373.13 \text{ kg/s}
\]

Verify:

None of the assumptions made in this problem solution can be verified.

Answers:

\[
m \quad 373 \text{ kg/s}
\]
Steam enters the turbine of a basic Rankine power cycle at a pressure of 10 MPa and a temperature $T_2$, and expands adiabatically to 6 kPa. The isentropic turbine efficiency is 85%. Saturated liquid water leaves the condenser at 6 kPa and the isentropic pump efficiency is 82%.

a.) For $T_2 = 580^\circ$C, determine the quality of the turbine effluent and the thermal efficiency of the cycle.

b.) Plot the quality of the turbine effluent and the thermal efficiency of the cycle for values of $T_2$ ranging from 580$^\circ$C to 700$^\circ$C at 10$^\circ$C increments.

Read: I used the TFT plug-in to solve this problem. It makes part (b) go much more quickly.

This is a straightforward application of the 1st Law, isentropic efficiency and the definition of the thermal efficiency of a power cycle.

**Given:**
- Water
- $P_2 = 10000$ kPa
- $P_3 = P_4 = 6$ kPa
- $\eta_{\text{turb}} = 85\%$
- $\eta_{\text{pump}} = 82\%$
- $x_4 = 0.0$ kg vap/kg

**Find:**

a.) Given:
- $T_2 = 580$ $^\circ$C
- $x_3 = ???$ kg vap/kg
- $\eta_{\text{th}} = ???$

b.) $T_2$ = (580, 600, 620, 640, 660, 680, 700) $^\circ$C
Plot $x_3$ and $\eta_{\text{th}}$ as a function of $T_2$.

**Assumptions:**
1. The turbine and pump are adiabatic.
2. No shaft work in the boiler or condenser.
3. The boiler and condenser are isobaric.
4. Changes in kinetic and potential energies are negligible.
5. Every process in the cycle operates at steady-state.
6. The condenser effluent is a saturated liquid.

**Diagram:**

![Diagram of the Superheat Rankine Cycle]
I like to organize the properties of the streams in more complex problems, like this one, in a table. This helps me keep track of what I know and what I do not know as I progress through the solution. The cells in the table below are color-coded.

In order to determine \( x_3 \), we will need to know \( H_3 \). In order to determine \( \eta_{th} \), we will need to determine \( W_{cycle} \) and \( Q_H \).

Because isentropic efficiencies were given for the pump and the turbine, it is safe to assume that these devices are adiabatic. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify the 1st Law as applied to the pump and turbine as follows.

Because we know the values of two intensive variables for state 2 (\( P_2 \) and \( T_2 \)) and for state 4 (\( P_4 \) and \( x_4 \)) we can immediately lookup the values of the other key intensive variables (\( H \) and \( S \)) for these two streams. For this problem, I am using the Thermal-Fluids Toolbox plug-in for Excel as a source for thermoodynamic properties.

### Table of Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>State 1</th>
<th>State 1S</th>
<th>State 2</th>
<th>State 3S</th>
<th>State 3</th>
<th>State 4</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>kPa</td>
</tr>
<tr>
<td>( T )</td>
<td>36.865</td>
<td>36.352</td>
<td>36.162</td>
<td>36.162</td>
<td>36.162</td>
<td>36.162</td>
<td>°C</td>
</tr>
<tr>
<td>( T )</td>
<td>310.01</td>
<td>309.50</td>
<td>309.31</td>
<td>309.31</td>
<td>309.31</td>
<td>309.31</td>
<td>K</td>
</tr>
<tr>
<td>( V )</td>
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<td>0.0010021</td>
<td>0.037286</td>
<td>19.226</td>
<td>21.390</td>
<td>0.0010071</td>
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</tr>
<tr>
<td>( U )</td>
<td>152.86</td>
<td>150.73</td>
<td>3202.5</td>
<td>1992.3</td>
<td>2199.4</td>
<td>151.03</td>
<td>kJ/kg</td>
</tr>
<tr>
<td>( H )</td>
<td>162.88</td>
<td>160.75</td>
<td>3575.3</td>
<td>2107.6</td>
<td>2327.77</td>
<td>151.03</td>
<td>kJ/kg</td>
</tr>
<tr>
<td>( S )</td>
<td>0.53</td>
<td>0.51903</td>
<td>6.8446</td>
<td>6.8446</td>
<td>7.56</td>
<td>0.51903</td>
<td>kJ/kg-K</td>
</tr>
<tr>
<td>( x )</td>
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<td>N/A</td>
<td>N/A</td>
<td>0.80988</td>
<td>0.90101</td>
<td>0</td>
<td>kg vap/kg</td>
</tr>
</tbody>
</table>

### Color codes:

- **Given**
- **TFT Lookup**
- **Main Calcs**
- **T convert**

### Equation 1

\[
\dot{Q}_{sh,pump} = \Delta H + \Delta E_{\text{kin}} + \Delta E_{\text{pot}}
\]

### Equation 2

\[
\eta_{th} = \frac{\dot{W}_{sh,pump}}{\dot{Q}_H} = \frac{\dot{W}_{sh,pump} + \dot{W}_{sh,turb}}{\dot{Q}_{12}}
\]

### Equation 3

\[
\dot{Q}_{23} = \dot{W}_{sh,turb} = \Delta H + \Delta E_{\text{kin}} + \Delta E_{\text{pot}}
\]

### Equation 4

\[
\dot{W}_{sh,pump} = \dot{H}_4 - \dot{H}_1
\]

### Equation 5

\[
\dot{W}_{sh,turb} = \dot{H}_2 - \dot{H}_3
\]
In order to determine $H_1$ and $H_3$, we will use the given isentropic efficiencies of the pump and turbine.

$$\eta_{S,\text{pump}} = \frac{-\hat{W}_{S,\text{isen}}}{-\hat{W}_{S,\text{act}}} = \frac{\hat{H}_4 - \hat{H}_{1S}}{\hat{H}_4 - \hat{H}_1} \quad \text{Eqn 10}$$

$$\eta_{S,\text{turb}} = \frac{-\hat{W}_{S,\text{act}}}{-\hat{W}_{S,\text{isen}}} = \frac{\hat{H}_2 - \hat{H}_3}{\hat{H}_2 - \hat{H}_{3S}} \quad \text{Eqn 11}$$

We can solve Eqns 10 & 11 for the unknowns $H_1$ and $H_3$.

$$\hat{H}_2 = \frac{\hat{H}_4 - \hat{H}_{1S}}{\eta_{S,\text{pump}}} \quad \text{Eqn 12}$$

$$\hat{H}_3 = \frac{\hat{H}_2 - \eta_{S,\text{turb}} (\hat{H}_2 - \hat{H}_{3S})}{\eta_{S,\text{turb}}} \quad \text{Eqn 13}$$

In order to use Eqns 12 & 13, we must first determine $H_{1S}$ and $H_{3S}$, the enthalpy of the effluent streams from the hypothetical isentropic compressor and turbine.

The keys here are that $S_{1S} = S_4$ & $P_{1S} = P_1$ and $S_{3S} = S_2$ & $P_{3S} = P_3$. Therefore, we know the values of two intensive variables at states $1S$ & $3S$ and we can use the Steam Tables (actually the TFT plug-in) to determine the values of $H_{1S}$ & $H_{3S}$.

$$S_{1S} = S_4 \quad 0.51903 \, \text{kJ/kg-K} \quad H_{1S} \quad 160.75 \, \text{kJ/kg}$$

$$S_{3S} = S_2 \quad 6.84459 \, \text{kJ/kg-K} \quad H_{3S} \quad 2107.6 \, \text{kJ/kg}$$

Now, we can plug values into Eqns 8, 9, 12 & 13 to evaluate $H_1$ and $H_3$.

$$W_{sh,turb,isan} = 1467.74 \, \text{kJ/kg} \quad W_{sh,pump,isan} = -9.72 \, \text{kJ/kg}$$

$$W_{sh,turb,act} = 1247.58 \, \text{kJ/kg} \quad W_{sh,pump,act} = -11.85 \, \text{kJ/kg}$$

$$H_3 = 2327.8 \, \text{kJ/kg} \quad H_1 = 162.88 \, \text{kJ/kg}$$

Now that we have $H_3$, we can determine $x_3$. Since $H_{\text{sat liq}} < H_3 < H_{\text{sat vap}}$, $x_3$ is defined and we can evaluate it using the saturation properties at $P_3 = 6 \, \text{kPa}$ and the following equation.

$$x_3 = \frac{\hat{H}_3 - \hat{H}_{\text{sat liq}}}{\hat{H}_{\text{sat vap}} - \hat{H}_{\text{sat liq}}} \quad \text{Eqn 14}$$

Plugging values into Eqn 14 yields:

$$x_3 \quad 0.90101 \, \text{kg vap/kg}$$

We still need to know $Q_{at}$ before we can use Eqn 5 to evaluate $\eta_{th}$.

We can evaluate $Q_{at} = Q_{12}$ by applying the 1st Law to the boiler.

$$\dot{Q}_{12} - \dot{W}_{sh,12} = \left[ \Delta \tilde{H} + \Delta \tilde{E}_{\text{kin}} + \Delta \tilde{E}_{\text{pot}} \right]_{12} \quad \text{Eqn 15}$$

Because the boiler has no moving parts, it is safe to assume that no shaft work crosses its boundary. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify the 1st Law as applied to the boiler.

$$\dot{Q}_{12} = \hat{H}_2 - \hat{H}_1 \quad \text{Eqn 16}$$

We know $H_1$ and $H_2$, so we can plug values into Eqn 16 to determine $Q_{at}$ and then use Eqn 1 to determine the thermal efficiency of the cycle.

$$Q_{at} = Q_{12} \quad 3412.5 \, \text{kJ/kg}$$

$$\eta_{th} \quad 36.21\%$$
Part b.) In this part of the problem, we just repeat all of the calculations in part (a) SIX more times!
Not every variable that we calculated in part (a) is different in part (b).
The variables that are different in part (b) because \( T_2 \) changes are the columns in the table, below.

<table>
<thead>
<tr>
<th>( T_2 ) (°C)</th>
<th>( S_2 ) (kJ/kg-K)</th>
<th>( H_{fg} ) (kJ/kg)</th>
<th>( H_2 ) (kJ/kg)</th>
<th>( W_{T,ben} ) (kJ/kg)</th>
<th>( W_{T,act} ) (kJ/kg)</th>
<th>( H_3 ) (kJ/kg)</th>
<th>( x_3 ) kg vap/kg</th>
<th>( \eta_{th} )</th>
<th>( \eta_{th} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>580</td>
<td>6.8446</td>
<td>2107.6</td>
<td>3575.3</td>
<td>1467.7</td>
<td>1247.6</td>
<td>2327.8</td>
<td>0.90101</td>
<td>0.362</td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>6.9020</td>
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<td>0.370</td>
<td></td>
</tr>
<tr>
<td>620</td>
<td>6.9578</td>
<td>2142.6</td>
<td>3674.2</td>
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<td>1301.8</td>
<td>2372.4</td>
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</tr>
<tr>
<td>640</td>
<td>7.0122</td>
<td>2159.5</td>
<td>3723.3</td>
<td>1563.8</td>
<td>1329.3</td>
<td>2394.0</td>
<td>0.92844</td>
<td>0.386</td>
<td></td>
</tr>
<tr>
<td>660</td>
<td>7.0653</td>
<td>2175.9</td>
<td>3772.3</td>
<td>1596.4</td>
<td>1357.0</td>
<td>2415.3</td>
<td>0.93726</td>
<td>0.394</td>
<td></td>
</tr>
<tr>
<td>680</td>
<td>7.1171</td>
<td>2191.9</td>
<td>3821.2</td>
<td>1629.3</td>
<td>1384.9</td>
<td>2436.3</td>
<td>0.94594</td>
<td>0.402</td>
<td></td>
</tr>
<tr>
<td>700</td>
<td>7.1679</td>
<td>2207.6</td>
<td>3870.1</td>
<td>1662.5</td>
<td>1413.1</td>
<td>2457.0</td>
<td>0.95449</td>
<td>0.411</td>
<td></td>
</tr>
</tbody>
</table>

The plot below shows that both the quality of the turbine effluent and the thermal efficiency of the cycle increase as the temperature of the turbine feed, \( T_2 \), increases. In fact, over this range, the relationships are both nearly linear.

We cannot keep increasing \( T_2 \) for ever because we would need to build the turbine out of materials that could function at ever higher temperatures. This becomes a material science problem!

Verify: None of the assumptions made in this problem solution can be verified.

Answers: a.) \( x_3 = 0.901 \) kg vap/kg  
          \( \eta_{th} = 36.2\% \)  
          b.) See the plot, above.
A power plant operates on a regenerative vapor power cycle with one closed feedwater heater. Steam enters the high-pressure turbine at 120 bar and 520°C and expands to 10 bar, where some of the steam is extracted and diverted to a closed feedwater heater. Condensate leaves the feedwater heater as a saturated liquid at 10 bar and then passes through an expansion valve before it is combined with the effluent from the low-pressure turbine. This combined stream flows to the condenser. The boiler feed leaves the feedwater heater at 120 bar and 170°C. The condenser pressure is 0.06 bar. Each turbine stage has an isentropic efficiency of 82%. The pump is essentially isentropic.

Determine ...

a.) The thermal efficiency of the cycle.

b.) The mass flow rate of water/steam through the boiler in kg/h. If the net power output of the cycle is 320 MW.

Read: I used the NIST Webbook for thermodynamic data to solve this problem.

This is a long and complicated problem because of the splitter, mixer and the closed feedwater heater.

In addition to the typical use of isentropic efficiency for the pump and turbines, we need to determine the fraction of the mass flow that flows through the LP turbine and the fraction that flows to the open FWH. The key is the FWH. The 1st Law applied to this process yields the mass flow fractions.

Once we know the mass flow fractions, we can determine thermal efficiency because \( m_1 \) drops out of this equation and only the mass flow fraction \( m_5/m_1 \) remains.

Part (b) is a straightforward application of the net power and the mass flow fractions to determine \( m_1 \). The hard part is part (a).

Given:

\[
\begin{array}{ccc}
P_N & 12000 & \text{kPa} \\
P_{\text{med}} & 1000 & \text{kPa} \\
P_{\text{low}} & 6 & \text{kPa} \\
T_1 & 170 & ^\circ\text{C} \\
T_2 & 520 & ^\circ\text{C} \\
\end{array}
\]

\[
\begin{array}{cc}
x_7 & 0 & \text{kg vap/kg} \\
x_{10} & 0 & \text{kg vap/kg} \\
\eta_{\text{turb}} & 82\% \\
\eta_{\text{pump}} & 100\% \\
W_{\text{cycle}} & 320000 & \text{kW} \\
\end{array}
\]

Find:

\[
\begin{array}{cc}
\eta_m & ??? \\
m_1 & ??? & \text{kg/h} \\
\end{array}
\]

Assumptions:

1 - Every process except the Boiler and condenser is adiabatic.

2 - Shaft work occurs only in the pump and the two turbines.

3 - Every process except the pump, turbines and expansion valve is isobaric.

4 - Changes in kinetic and potential energies are negligible.

5 - Every process in the cycle operates at steady-state.

6 - The condenser effluent is a saturated liquid.
Diagram:

Equations / Data / Solve:

I like to organize the properties of the streams in more complex problems, like this one, in a table. This helps me keep track of what I know and what I do not know as I progress through the solution. The cells in the table below are color-coded.

<table>
<thead>
<tr>
<th>Stream</th>
<th>P</th>
<th>T</th>
<th>H</th>
<th>S</th>
<th>x</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12000</td>
<td>170</td>
<td>725.31</td>
<td>2.0276</td>
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<td>Sub Liq.</td>
</tr>
<tr>
<td>2</td>
<td>12000</td>
<td>520</td>
<td>3403.4</td>
<td>6.5585</td>
<td>N/A</td>
<td>Super. Vap.</td>
</tr>
<tr>
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<td>2765.1</td>
<td>6.5585</td>
<td>0.9940</td>
<td>VLE</td>
</tr>
<tr>
<td>3, 4, 5</td>
<td>1000</td>
<td>2880.0</td>
<td>2880.0</td>
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<td>Super. Vap.</td>
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<tr>
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<tr>
<td>6</td>
<td>6</td>
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</tr>
<tr>
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<td>0</td>
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<td>163.51</td>
<td>0.52082</td>
<td>N/A</td>
<td></td>
</tr>
</tbody>
</table>

Color Codes:

| Given | Isen Comp | Isen Turb | Real Comp | Real Turb | Interpolate | Interpolate | Interpolate | Interpolate |

Part a.) We can determine the thermal efficiency of this power cycle from its definition.

\[
\eta_{th} = \frac{\dot{W}_{sh,cycle}}{\dot{Q}_{li}} = \frac{\dot{m}_1 \dot{W}_{sh,pump} + \dot{m}_1 \dot{W}_{sh,HP} + \dot{m}_5 \dot{W}_{sh,LP}}{\dot{m}_1 \dot{Q}_{li}} \tag{Eqn 1}
\]

We can determine each of the work terms in Eqn 1 as well as \( Q_{li} \) by applying the 1st Law to the pump, the two turbines and the boiler.

\[
\dot{Q} - \dot{W}_{sh} = \Delta \dot{H} + \Delta \dot{E}_{kin} + \Delta \dot{E}_{pot} \tag{Eqn 2}
\]

We can assume the turbines are adiabatic because we are given values of the isentropic efficiency for them. In the absence of heat transfer data for the pump, we can assume that it is also adiabatic. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify the 1st Law as applied to the pump, turbines and boiler as follows.

\[
\dot{W}_{sh,pump} = \dot{H}_{10} - \dot{H}_{11} \tag{Eqn 3}
\]

\[
\dot{W}_{sh,HP} = \dot{H}_2 - \dot{H}_3 \tag{Eqn 5}
\]

\[
\dot{Q}_{li} = \dot{H}_2 - \dot{H}_1 \tag{Eqn 4}
\]

\[
\dot{W}_{sh,LP} = \dot{H}_3 - \dot{H}_6 \tag{Eqn 6}
\]
Before we can use **Eqns 1 & 3 - 6**, we need to determine the $H$ for almost every stream in the cycle! We can start by looking up the values for the stream for which we already know the values of two intensive variables: streams 1, 2, 7 & 10.

<table>
<thead>
<tr>
<th>Stream</th>
<th>$H$ (kJ/kg)</th>
<th>$S$ (kJ/kg-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>725.31</td>
<td>2.0276</td>
</tr>
<tr>
<td>$H_2$</td>
<td>3403.4</td>
<td>6.5585</td>
</tr>
<tr>
<td>$H_7$</td>
<td>762.52</td>
<td>2.1381</td>
</tr>
<tr>
<td>$H_{10}$</td>
<td>151.48</td>
<td>0.52082</td>
</tr>
</tbody>
</table>

Plugging values into **Eqn 4** yields:

$$Q_{12} = Q_H = 2678.1 \text{ kJ/kg}$$

In order to determine $H_3$, $H_6$ & $H_{11}$, we must analyze hypothetical isentropic turbines and make use of the isentropic efficiency of each process.

For the isentropic pump & the hypothetical isentropic high-pressure turbine:

<table>
<thead>
<tr>
<th>Stream</th>
<th>$S_{3S}$</th>
<th>$S_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{3S}$</td>
<td>6.5585</td>
<td>0.52082</td>
</tr>
</tbody>
</table>

Now we can use **NIST Webbook** data to determine $H_{3S}$ and $H_{11}$.

At 1 MPa:

<table>
<thead>
<tr>
<th>Stream</th>
<th>$H_{sat _liq}$</th>
<th>$H_{sat _vap}$</th>
<th>$S_{sat _liq}$</th>
<th>$S_{sat _vap}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{sat _liq}$</td>
<td>762.52</td>
<td>21381</td>
<td>2.1381</td>
<td></td>
</tr>
<tr>
<td>$H_{sat _vap}$</td>
<td>2777.1</td>
<td>6.5850</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since $S_{3S}$ lies between $S_{sat \_liq}$ and $S_{sat \_vap}$, we must determine the quality, $x_{3S}$:

$$x_{3S} = \frac{S_{3S} - S_{sat \_liq}}{S_{sat \_vap} - S_{sat \_liq}} \quad \text{Eqn 7}$$

Plugging values into **Eqn 7** yields:

$$x_{3S} = 0.99404 \text{ kg vap/kg}$$

Plugging values into **Eqn 8** yields:

<table>
<thead>
<tr>
<th>Stream</th>
<th>$H_{3S}$</th>
<th>$T_{3S}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{3S}$</td>
<td>2765.1</td>
<td>179.9 $^\circ\text{C}$</td>
</tr>
</tbody>
</table>

At 12 MPa:

<table>
<thead>
<tr>
<th>Stream</th>
<th>$H_{sat _liq}$</th>
<th>$H_{sat _vap}$</th>
<th>$S_{sat _liq}$</th>
<th>$S_{sat _vap}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{sat _liq}$</td>
<td>1491.5</td>
<td>34967</td>
<td>2.1381</td>
<td></td>
</tr>
<tr>
<td>$H_{sat _vap}$</td>
<td>2685.4</td>
<td>5.4939</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since $S_{11} < S_{sat \_liq}$, stream 11 is a subcooled liquid and we must interpolate on **NIST Webbook** data to determine $H_{11}$.

<table>
<thead>
<tr>
<th>$T$ ($^\circ\text{C}$)</th>
<th>$H$ (kJ/kg)</th>
<th>$S$ (kJ/kg-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>161.52</td>
<td>0.51438</td>
</tr>
<tr>
<td>36.48</td>
<td>163.51</td>
<td>0.52082</td>
</tr>
<tr>
<td>37</td>
<td>165.67</td>
<td>0.52778</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stream</th>
<th>$T_{11}$</th>
<th>$S_{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{11}$</td>
<td>36.48 $^\circ\text{C}$</td>
<td>63.51 kJ/kg-K</td>
</tr>
<tr>
<td>$S_{11}$</td>
<td>5.4939</td>
<td></td>
</tr>
</tbody>
</table>

Plugging values into **Eqn 3** yields:

$$W_{P\_act} = -12.034 \text{ kJ/kg}$$

Now, we must use the isentropic efficiency of the high-pressure turbine to determine $H_3$.

$$\eta_{S,HP} = \frac{-\dot{W}_{Sh,act}}{-\dot{W}_{Sh,isen}} = \frac{\hat{H}_2 - \hat{H}_3}{\hat{H}_2 - \hat{H}_{3S}} \quad \text{Eqn 9}$$

Solving **Eqn 9** for $H_3$ yields:

$$\hat{H}_3 = \hat{H}_2 - \eta_{S,turb} (\hat{H}_2 - \hat{H}_{3S}) \quad \text{Eqn 10}$$

Plugging values into **Eqns 9 & 10** yields:

| $W_{HP,isen}$ | 638.31 kJ/kg |
| $W_{HP,act}$  | 523.41 kJ/kg |
| $H_3$         | 2880.0 kJ/kg |
Since $H_3 > H_{\text{sat vap}}$ at 1 MPa, stream 3 is superheated steam. We must interpolate on entropy data for superheated steam from the NIST Webbook in order to determine $S_3$.

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>H (kJ/kg)</th>
<th>S (kJ/kg-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>221</td>
<td>2877.8</td>
<td>6.7980</td>
</tr>
<tr>
<td>221.95</td>
<td>2879.99</td>
<td>6.8025</td>
</tr>
<tr>
<td>222</td>
<td>2880.1</td>
<td>6.8027</td>
</tr>
</tbody>
</table>

Next we need to analyze the hypothetical low-pressure turbine. $S_{6S} = S_3$.

At 6 kPa:

- $H_{\text{sat liq}} = 151.48$ kJ/kg
- $H_{\text{sat vap}} = 2566.6$ kJ/kg
- $S_{\text{sat liq}} = 0.5208$ kJ/kg-K
- $S_{\text{sat vap}} = 8.3290$ kJ/kg-K

Since $S_{6S}$ lies between $S_{\text{sat liq}}$ and $S_{\text{sat vap}}$, we must determine the quality, $x_{6S}$.

\[
x_{6S} = \frac{S_{6S} - S_{\text{sat liq}}}{S_{\text{sat vap}} - S_{\text{sat liq}}} \quad \text{Eqn 11}
\]

Plugging values into Eqn 7 yields:

\[
\hat{H}_{6S} = x_{6S} \hat{H}_{\text{sat vap}} + (1 - x_{6S}) \hat{H}_{\text{sat liq}} \quad \text{Eqn 12}
\]

Plugging values into Eqn 8 yields:

\[
H_{6S} = 2094.4 \text{ kJ/kg} \quad T_{6S} = T_{\text{sat}} = 36.16 \text{ °C} \quad \text{Eqn 11}
\]

Now, we must use the isentropic efficiency of the low-pressure turbine to determine $H_6$.

\[
\eta_{S,LP} = -\frac{\dot{W}_{\text{sh,act}}}{\dot{W}_{\text{sh,isen}}} = -\frac{\hat{H}_3 - \hat{H}_6}{\hat{H}_3 - \hat{H}_{6S}} \quad \text{Eqn 11}
\]

Solving Eqn 11 for $H_6$ yields:

\[
\hat{H}_6 = \hat{H}_3 - \eta_{S,\text{turb}} \left( \hat{H}_3 - \hat{H}_{6S} \right) \quad \text{Eqn 12}
\]

Plugging values into Eqns 9 & 10 yields:

\[
W_{\text{LP,isen}} = 785.55 \text{ kJ/kg} \quad W_{\text{LP,act}} = 644.15 \text{ kJ/kg} \quad H_6 = 2235.8 \text{ kJ/kg} \quad \text{Eqn 13}
\]

If we assume the expansion valve is adiabatic and involves no shaft work and changes in kinetic and potential energies are negligible, then the 1st Law reduces to: $H_6 = H_7$.

We can determine $H_9$ by applying the steady-state, MIMO form of the 1st law to the Mixer.

\[
\dot{Q} - \dot{W} = \sum_{j=1}^{\#\text{outlets}} \dot{m}_{\text{out},j} \left[ \hat{H}_{\text{out}} + \hat{E}_{\text{kin,out}} + \hat{E}_{\text{pot,out}} \right] - \sum_{i=1}^{\#\text{inlets}} \dot{m}_{\text{in},i} \left[ \hat{H}_{\text{in}} + \hat{E}_{\text{kin,in}} + \hat{E}_{\text{pot,in}} \right] \quad \text{Eqn 13}
\]

Assume that the mixer is adiabatic, no shaft work is involved and changes in kinetic and potential energies are negligible.

\[
\dot{m}_4 \hat{H}_8 + \dot{m}_5 \hat{H}_6 = \dot{m}_1 \hat{H}_9 \quad \text{Eqn 14}
\]

Solving Eqn 14 for $H_9$ yields:

\[
\hat{H}_9 = \frac{\dot{m}_4}{\dot{m}_1} \hat{H}_8 + \frac{\dot{m}_5}{\dot{m}_1} \hat{H}_6 \quad \text{Eqn 15}
\]

We must apply the 1st law to the Closed Feedwater Heater in order to determine the fraction of the flow of stream 1 that goes to the LP turbine and the fraction that goes to the FWH.

\[
\dot{Q} - \dot{W} = \sum_{j=1}^{\#\text{outlets}} \dot{m}_{\text{out},j} \left[ \hat{H}_{\text{out}} + \hat{E}_{\text{kin,out}} + \hat{E}_{\text{pot,out}} \right] - \sum_{i=1}^{\#\text{inlets}} \dot{m}_{\text{in},i} \left[ \hat{H}_{\text{in}} + \hat{E}_{\text{kin,in}} + \hat{E}_{\text{pot,in}} \right] \quad \text{Eqn 16}
\]
With the same assumptions that were made for the mixer, Eqn 16 simplifies to:

$$m4 \dot{H}4 + m1 \dot{H}11 = m1 \dot{H}1 + m4 \dot{H}7$$

Eqn 17

Now, we can solve Eqn 17 for $m4/m1$, the fraction of stream 1 diverted to the FWH.

$$m1 \left( \dot{H}1 - \dot{H}11 \right) = m4 \left( \dot{H}4 - \dot{H}7 \right)$$

Eqn 18

$$\frac{m4}{m1} = \frac{\dot{H}4 - \dot{H}7}{\dot{H}1 - \dot{H}11}$$

Eqn 19

Plugging values into Eqn 19 yields:

$$m4/m1 = \frac{0.2653}{0.2653}$$

The mass balance on the splitter is:

$$m1 = m4 + m5$$

Eqn 20

Divide Eqn 20 by $m1$ and solve for $m5/m1$:

$$\frac{m4}{m1} + \frac{m5}{m1} = 1$$

Eqn 21

$$\frac{m5}{m1} = 1 - \frac{m4}{m1}$$

Eqn 22

Plugging values into Eqn 22 yields:

$$m5/m1 = \frac{0.7347}{0.7347}$$

Now, we can plug values back into Eqn 15 to evaluate $H9$.

$$H9 = 1844.9 \text{ kJ/kg}$$

Finally, we can rearrange Eqn 1 to put in terms of the mass flow fractions, $m4/m1$ and $m5/m1$:

$$\eta_{th} = \frac{\dot{W}_{sh, cycle} + \dot{W}_{sh, HP} + \frac{m5}{m1} \dot{W}_{sh, LP}}{\dot{Q}_{hf}}$$

Eqn 23

Plugging values into Eqn 23 yields:

$$\eta_{th} = 36.77\%$$

Part b.) The key to determining $m1$ is the net power output of the cycle.

$$\dot{W}_{sh, cycle} = m1 \dot{W}_{sh, pump} + m1 \dot{W}_{sh, HP} + m5 \dot{W}_{sh, LP} = 320,000 \text{ kW}$$

Eqn 24

Solving Eqn 24 for $m1$ yields:

$$m1 = \frac{\dot{W}_{sh, cycle}}{\dot{W}_{sh, pump} + \dot{W}_{sh, HP} + \frac{m5}{m1} \dot{W}_{sh, LP}} = \frac{320,000 \text{ kW}}{320,000 \text{ kW}}$$

Eqn 25

Plugging values into Eqn 25 yields:

$$m1 = \frac{325.00 \text{ kg/s}}{325.00 \text{ kg/s}}$$

Finally, using the mass flow ratios determined in part (a) yields:

$m4 = 86.2 \text{ kg/s}$

$m5 = 238.8 \text{ kg/s}$

Most of the steam goes through the LP turbine. Only about 27% is used to preheat the boiler feed in the closed feedwater heater.

Verify: None of the assumptions made in this problem solution can be verified.

Answers: a.) $\eta_{th} = 36.8\%$ b.) $m1 = 325 \text{ kg/s}$
The diagram shows a two-stage, vapor-compression refrigeration system that uses ammonia as the working fluid. The system uses a flash drum to achieve intercooling. The evaporator has a refrigeration capacity of 30 tons and produces a saturated vapor effluent at -20°F. In the first compressor stage, the refrigerant is compressed adiabatically to 80 psia, which is the pressure in the mixer. Saturated vapor at 80 psia enters the second compressor stage and is compressed adiabatically to 250 psia. Each compressor stage has an isentropic efficiency of 85%. There are no significant pressure drops as the refrigerant passes through the heat exchangers. Saturated liquid enters each expansion valve.

Determine...

a.) The mass flow rate of ammonia through each compressor in \( \text{lbm/h} \).
b.) The power input to each compressor in Btu/h.
c.) The coefficient of performance of the cycle.

Read:

In addition to the typical use of isentropic compressor efficiencies and isenthalpic expansion valves, the key to this problem is the flash drum. The stream leaving the top of the flash drum, stream 6, is a saturated vapor and the stream leaving the bottom of the flash drum, stream 4, is a saturated liquid.

All the heat given up in the flash drum by the lower cycle is absorbed by the upper cycle. As a result, the 1st law applied to the flash drum provides the connection between the mass flow rate of ammonia in the lower cycle to the mass flow rate of ammonia in the upper cycle. Once we use the given refrigeration capacity, \( Q_C \), to determine the flow rate of ammonia in the lower cycle, we can use the 1st Law applied to the flash drum to determine the mass flow rate of ammonia in the upper cycle.

I used the NIST Webbook to obtain the thermodynamic properties of ammonia.

Given:

<table>
<thead>
<tr>
<th>( Q_C )</th>
<th>30</th>
<th>tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Ton</td>
<td>200</td>
<td>Btu/min</td>
</tr>
<tr>
<td>QC</td>
<td>100</td>
<td>Btu/s</td>
</tr>
<tr>
<td>360000</td>
<td>Btu/h</td>
<td></td>
</tr>
<tr>
<td>( P_N )</td>
<td>250</td>
<td>psia</td>
</tr>
<tr>
<td>( P_{med} )</td>
<td>80</td>
<td>psia</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>-20</td>
<td>°F</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( \eta_{S,comp} )</td>
<td>85%</td>
<td></td>
</tr>
<tr>
<td>( x_4 )</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( x_5 )</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( x_6 )</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Find:

a.) \( m_1 \) | ??? | \( \text{lb}_m/\text{lb}_m \) |
| \( m_2 \) | ??? | \( \text{lb}_m/\text{lb}_m \) |

b.) \( W_{c1,act} \) | ??? | Btu/h |
| \( W_{c2,act} \) | ??? | Btu/h |

c.) COPR | ??? |
**Assumptions:**

1. Every process is adiabatic except the Evaporator and Condenser.
2. Shaft work occurs only in the two compressors.
3. Every process except the pump, turbines and expansion valve is isobaric.
4. Changes in kinetic and potential energies are negligible.
5. Every process in the cycle operates at steady-state.
6. The condenser effluent is a saturated liquid.

**Equations / Data / Solve:**

I like to organize the properties of the streams in more complex problems, like this one, in a table. This helps me keep track of what I know and what I do not know as I progress through the solution. The cells in the table below are color-coded.

<table>
<thead>
<tr>
<th>State</th>
<th>P (psia)</th>
<th>T (°F)</th>
<th>H (Btu/lbm)</th>
<th>S (Btu/lbm °R)</th>
<th>x (lbm vap/lbm)</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.279</td>
<td>-20</td>
<td>91.56</td>
<td>0.20929</td>
<td>0.1205</td>
<td>Sat'd Mix</td>
</tr>
<tr>
<td>2</td>
<td>18.279</td>
<td>-20</td>
<td>91.56</td>
<td>0.20929</td>
<td>0.1205</td>
<td>Sat'd Mix</td>
</tr>
<tr>
<td>3S</td>
<td>80</td>
<td>159.11</td>
<td>692.30</td>
<td>1.3766</td>
<td>N/A</td>
<td>Super. Vap.</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>186.55</td>
<td>707.75</td>
<td>1.4011</td>
<td>N/A</td>
<td>Super. Vap.</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>44.391</td>
<td>91.56</td>
<td>0.19795</td>
<td>0.1433</td>
<td>Sat'd Mix</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>44.391</td>
<td>91.56</td>
<td>0.19795</td>
<td>0.1433</td>
<td>Sat'd Mix</td>
</tr>
<tr>
<td>6</td>
<td>80</td>
<td>44.391</td>
<td>91.56</td>
<td>0.19795</td>
<td>0.1433</td>
<td>Sat'd Mix</td>
</tr>
<tr>
<td>7S</td>
<td>250</td>
<td>193.80</td>
<td>693.91</td>
<td>1.2539</td>
<td>N/A</td>
<td>Super. Vap.</td>
</tr>
<tr>
<td>7</td>
<td>250</td>
<td>212.84</td>
<td>706.27</td>
<td>1.2726</td>
<td>N/A</td>
<td>Super. Vap.</td>
</tr>
<tr>
<td>8</td>
<td>250</td>
<td>110.75</td>
<td>167.81</td>
<td>0.33842</td>
<td>0.1433</td>
<td>Sat'd Mix</td>
</tr>
</tbody>
</table>

Color codes:
- Given
- Isentropic
- Interp.
- Interp.
- Interp.
- Interp.
- Interp.
- Interp.

I also like to make a table of all the relevant saturation properties.

<table>
<thead>
<tr>
<th>Psat psia</th>
<th>Tsat °F</th>
<th>Hsat,liq Btu/lbm</th>
<th>Hsat,vap Btu/lbm</th>
<th>Ssat,liq Btu/lbm °R</th>
<th>Ssat,vap Btu/lbm °R</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.279</td>
<td>-20.000</td>
<td>21.253</td>
<td>91.56</td>
<td>0.049391</td>
<td>1.3766</td>
</tr>
<tr>
<td>80</td>
<td>44.391</td>
<td>159.11</td>
<td>692.30</td>
<td>1.3766</td>
<td>N/A</td>
</tr>
<tr>
<td>250</td>
<td>110.75</td>
<td>159.11</td>
<td>692.30</td>
<td>1.3766</td>
<td>N/A</td>
</tr>
</tbody>
</table>

**Part a.)**

The key to determining m1 and m5 is the given value of the refrigeration load, \( Q_C = Q_{12} \).

The importance of the value of \( Q_C \) may become more clear by applying the steady-state form of the 1st Law of open systems to the Evaporator.

\[
\dot{Q}_{12} - \dot{W}_{sh} = \dot{m}_1 \left[ \Delta H + \Delta E_{\text{kin}} + \Delta E_{\text{pot}} \right]_{12}
\]  

**Eqn 1**

The shaft work for a HEX is zero because there are no moving parts. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify **Eqn 1** as follows.

\[
\dot{Q}_{12} = \dot{m}_1 \left( \hat{H}_2 - \hat{H}_1 \right)
\]  

**Eqn 2**

We can now solve **Eqn 2** for \( \dot{m}_1 \):

\[
\dot{m}_1 = \frac{\dot{Q}_{12}}{\hat{H}_2 - \hat{H}_1}
\]  

**Eqn 3**

In order to use **Eqn 3**, we need to know \( \hat{H}_1 \) and \( \hat{H}_2 \). We can lookup \( \hat{H}_2 \) in the NIST Webbook because we know the values of two intensive variables for state 2: \( T_2 \) and \( x_2 \).

\[
\hat{H}_2 = 604.79 \ \text{Btu/lbm}
\]

We can determine \( \hat{H}_1 \) from \( \hat{H}_4 \) by applying the steady-state form of the 1st Law for open systems to the expansion valve in the lower cycle. Since there is no shaft work at the valve, we assume it is adiabatic and that changes in kinetic and potential energies are negligible. The valve is isenthalpic: \( \hat{H}_1 = \hat{H}_4 \). We can lookup \( \hat{H}_4 \) in the NIST Webbook because we know the values of two intensive variables for state 4: \( T_4 \) and \( x_4 \).

\[
\hat{H}_4 = 91.56 \ \text{Btu/lbm}
\]

Now, we can plug values into **Eqn 3** to determine \( \dot{m}_1 \).
The key to determining \( m_5 \) is that all the heat given up in the flash drum by the lower cycle is absorbed by the upper cycle. So, we need to apply the steady-state, MIMO form of the 1st Law for open systems to the flash drum.

\[
\dot{Q} - \dot{W}_k = \sum_{j=1}^{\# \text{inlets}} \dot{m}_{\text{out},j} \left[ \dot{H}_{\text{out}} + \dot{H}_{\text{kin, out}} + \dot{H}_{\text{pot, out}} \right] - \sum_{i=1}^{\# \text{outlets}} \dot{m}_{\text{in},i} \left[ \dot{H}_{\text{in}} + \dot{H}_{\text{kin, in}} + \dot{H}_{\text{pot, in}} \right] \tag{Eqn 4}
\]

Assume that the flash drum is adiabatic, no shaft work is involved and changes in kinetic and potential energies are negligible. Then, **Eqn 4** simplifies to:

\[
\dot{m}_1 \dot{H}_3 + \dot{m}_3 \dot{H}_5 = \dot{m}_1 \dot{H}_4 + \dot{m}_3 \dot{H}_6 \tag{Eqn 5}
\]

We can solve **Eqn 5** for the unknown flow rate, \( m_5 \). 

\[
\dot{m}_5 = \frac{\dot{m}_1 \dot{H}_4 - \dot{H}_3}{\dot{H}_2 - \dot{H}_6} \tag{Eqn 6}
\]

In order to use **Eqn 6**, we must first determine \( H_2, H_3 & H_6 \). Let's start with \( H_6 \) because it is easiest. We know \( P_6 = 80 \) psia and we know it is a saturated vapor. So, we can lookup \( H_6 \) in the NIST Webbook without interpolation.

\[
H_6 = 623.83 \text{ Btu/lbm} \]

Next, let's work on getting \( H_5 \). The expansion valve in the upper cycle is isenthalpic for the same reasons mentioned above in the analysis of the expansion valve in the lower cycle. Therefore, \( H_5 = H_8 \). We can lookup \( H_8 \) in the NIST Webbook because we know \( P_8 = 250 \) psia and we know that stream 8 is a saturated liquid.

\[
H_5 = H_8 = 167.81 \text{ Btu/lbm}
\]

In order to determine \( H_3 \) and \( H_7 \), we will use the given isentropic efficiencies for the two compressors.

\[
\eta_{S, c1} = \frac{-\dot{W}_{\text{Sh, isen}}}{-\dot{W}_{\text{Sh, act}}} = \frac{\dot{H}_2 - \dot{H}_{3S}}{\dot{H}_2 - \dot{H}_3} \tag{Eqn 7}
\]

\[
\eta_{S, c2} = \frac{-\dot{W}_{\text{Sh, isen}}}{-\dot{W}_{\text{Sh, act}}} = \frac{\dot{H}_6 - \dot{H}_{7S}}{\dot{H}_6 - \dot{H}_7} \tag{Eqn 8}
\]

We can solve **Eqns 7 & 8** for the unknowns \( H_3 \) and \( H_7 \).

\[
\dot{H}_3 = \dot{H}_2 - \frac{\dot{H}_2 - \dot{H}_{3S}}{\eta_{S, c1}} \tag{Eqn 9}
\]

\[
\dot{H}_7 = \dot{H}_6 - \frac{\dot{H}_6 - \dot{H}_{7S}}{\eta_{S, c2}} \tag{Eqn 10}
\]

In order to use **Eqns 9 & 10**, we must first determine \( H_{3S} \) and \( H_{7S} \), the enthalpy of the effluent streams from the hypothetical isentropic compressors.

The keys here are that \( S_{3S} = S_2 \) & \( P_{3S} = P_3 \) and \( S_{7S} = S_6 \) & \( P_{7S} = P_7 \). Therefore, we know the values of two intensive variables at states 3S & 7S and we can use the NIST Webbook to determine the values of \( H_{3S} \) & \( H_{7S} \).

\[
S_{3S} = S_2 = 1.3766 \text{ kJ/kg-K} \quad S_{7S} = S_6 = 1.2539 \text{ kJ/kg-K}
\]

<table>
<thead>
<tr>
<th>( T ) (°F)</th>
<th>( H ) (Btu/lbm)</th>
<th>( S ) (Btu/lbm°R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>159</td>
<td>692.24</td>
<td>1.3765</td>
</tr>
<tr>
<td>160</td>
<td>692.80</td>
<td>1.3774</td>
</tr>
<tr>
<td>193</td>
<td>693.38</td>
<td>1.2531</td>
</tr>
<tr>
<td>194</td>
<td>694.04</td>
<td>1.2541</td>
</tr>
</tbody>
</table>

### Notes

<table>
<thead>
<tr>
<th>( T_{3S} )</th>
<th>( H_{3S} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>159.11 °F</td>
<td>692.30 kJ/kg</td>
</tr>
</tbody>
</table>
Now, we can plug values into Eqns 7 - 10 & 6 to evaluate $H_2$ and $H_7$ & $m_5$.

$W_{c1,lsen}$ = -70.08 Btu/lbm
$W_{c1,act}$ = -82.44 Btu/lbm
$H_7$ = 706.27 Btu/lbm

$W_{c2,lsen}$ = -87.51 Btu/lbm
$W_{c2,act}$ = -102.96 Btu/lbm
$H_3$ = 707.75 Btu/lbm

$m_5$ = 948 lbm/h

**Part b.**
In the process of completing part (a), we have already determined the answers to part (b)!

$W_{c1,act}$ = -72217 Btu/h
$W_{c2,act}$ = -78141 Btu/h

**Part c.**
We can determine $COP_R$ from its definition.

$$COP_R = \frac{\dot{Q}_C}{-\dot{W}_{cycle}} = \frac{\dot{Q}_{12}}{-\dot{m}_1\dot{W}_{sh,c1} + \dot{m}_s\dot{W}_{sh,c2}}$$

Eqn 11

Plugging values into Eqns 12 & 11 yields:

-Wcycle = 150358 Btu/h

$COP_R$ = 2.39

**Verify:**
None of the assumptions made in this problem solution can be verified.

**Answers:**

a.) $m_1$ = 701 lbm/h $m_5$ = 948 lbm/h

b.) $W_{c1,act}$ = -72217 Btu/h $W_{c2,act}$ = -78141 Btu/h

c.) $COP_R$ = 2.39

**Optional:**
Here are a couple of additional, optional interpolations.

<table>
<thead>
<tr>
<th>T (°F)</th>
<th>H (Btu/lbm)</th>
<th>S (Btu/lbm °R)</th>
<th>T (°F)</th>
<th>H (Btu/lbm)</th>
<th>S (Btu/lbm °R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>212</td>
<td>705.74</td>
<td>1.2718</td>
<td>186</td>
<td>707.44</td>
<td>1.4006</td>
</tr>
<tr>
<td>212.84</td>
<td>706.27</td>
<td>1.2726</td>
<td>186.55</td>
<td>707.75</td>
<td>1.4011</td>
</tr>
<tr>
<td>213</td>
<td>706.38</td>
<td>1.2727</td>
<td>187</td>
<td>708.00</td>
<td>1.4015</td>
</tr>
</tbody>
</table>
A vapor-compression heat pump uses R-134a as the working fluid. The refrigerant enters the compressor at 2.4 bar and 0°C at a volumetric flow rate of 0.60 m³/min. Compression is adiabatic to 9 bar and 60°C and saturated liquid leaves the condenser at 9 bar. Determine...

a.) The power input to the compressor in kW.
b.) The heating capacity of the heat pump in kW.
c.) The coefficient of performance.
d.) The isentropic compressor efficiency.

Read: This is a straightforward heat pump problem. I used the NIST Webbook to obtain the thermodynamic properties of R-134a.

The keys to part (a) are applying the 1st law to the compressor and determining the mass flow rate from the volumetric flow rate using the specific volume.

The key to part (b) is applying the 1st law to the condenser.

Part (c) is a straightforward application of the definition of the COP for a heat pump.

In part (d) we must determine the enthalpy of the effluent stream from a hypothetical, isentropic compressor. Then, we can use this value to determine the isentropic efficiency of our actual compressor.

Given:

<table>
<thead>
<tr>
<th></th>
<th>P_2 = P_1</th>
<th>240</th>
<th>kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T_2</td>
<td>0</td>
<td>°C</td>
</tr>
<tr>
<td></td>
<td>V_2</td>
<td>0.60</td>
<td>m³/min</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>P_3 = P_4</th>
<th>900</th>
<th>kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T_3</td>
<td>60</td>
<td>°C</td>
</tr>
<tr>
<td></td>
<td>x_4</td>
<td>0.0</td>
<td>kg vap/kg</td>
</tr>
</tbody>
</table>

Find:

a.) W_{th,comp} ??? kW
b.) Q_H = -Q_{34} ??? kW
c.) COP_{HP} ???
d.) \eta_{S,comp} ???

Diagram:

Assumptions:

1 - The compressor is adiabatic.
2 - No shaft work in the evaporator or condenser.
3 - The evaporator and condenser are isobaric.
4 - Changes in kinetic and potential energies are negligible.
5 - Every process in the cycle operates at steady-state.
6 - The condenser effluent is a saturated liquid.
I like to organize the properties of the streams in more complex problems, like this one, in a table. This helps me keep track of what I know and what I do not know as I progress through the solution. The cells in the table below are color-coded.

<table>
<thead>
<tr>
<th>Property</th>
<th>State 1</th>
<th>State 2</th>
<th>State 3S</th>
<th>State 3</th>
<th>State 4</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>240</td>
<td>240</td>
<td>900</td>
<td>900</td>
<td>900</td>
<td>kPa</td>
</tr>
<tr>
<td>T</td>
<td>0</td>
<td>45.676</td>
<td>60</td>
<td>35.526</td>
<td></td>
<td>°C</td>
</tr>
<tr>
<td>V</td>
<td>249.78</td>
<td>400.11</td>
<td>428.38</td>
<td>443.28</td>
<td>249.78</td>
<td>m³/kg</td>
</tr>
<tr>
<td>H</td>
<td>1.7475</td>
<td>1.7475</td>
<td>1.7932</td>
<td>1.1695</td>
<td></td>
<td>kJ/kg-K</td>
</tr>
<tr>
<td>x</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>0</td>
<td>kg vap/kg</td>
</tr>
<tr>
<td>Phase</td>
<td>Sat'd Mix</td>
<td>Super Vap</td>
<td>Super Vap</td>
<td>Super Vap</td>
<td>Sat Liquid</td>
<td></td>
</tr>
</tbody>
</table>


I also like to make a table of all the relevant saturation properties.

<table>
<thead>
<tr>
<th>P_sat</th>
<th>T_sat</th>
<th>V_sat</th>
<th>H_sat</th>
<th>S_sat</th>
<th>V_sat</th>
<th>H_sat</th>
<th>S_sat</th>
</tr>
</thead>
<tbody>
<tr>
<td>kPa</td>
<td>°C</td>
<td>m³/kg</td>
<td>kJ/kg</td>
<td>kJ/kg</td>
<td>kJ/kg</td>
<td>kJ/kg-K</td>
<td>kJ/kg-K</td>
</tr>
<tr>
<td>240</td>
<td>-5.3653</td>
<td>7.6202E-04</td>
<td>0.083906</td>
<td>192.65</td>
<td>395.44</td>
<td>0.97364</td>
<td>1.7303</td>
</tr>
<tr>
<td>900</td>
<td>35.526</td>
<td>8.5811E-04</td>
<td>0.022687</td>
<td>249.78</td>
<td>417.4</td>
<td>1.1695</td>
<td>1.7126</td>
</tr>
</tbody>
</table>

Part a.) We can determine the shaft work for the compressor by applying the steady-state form of the 1st Law for open systems.

\[ Q_{23} - W_{sh,comp} = \dot{m} \left[ \Delta H + \Delta E_{kin} + \Delta E_{pot} \right]_{23} \]  

**Eqn 1**

Because we are asked to determine the isentropic efficiency of the compressor in part (d), it is safe to assume that the compressor is adiabatic. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify **Eqn 1**.

\[ \dot{W}_{sh,comp} = \dot{m} \left( \hat{H}_2 - \hat{H}_3 \right) \]  

**Eqn 2**

We can immediately look up the properties of streams 2 & 3 because I know the values of two intensive variables for each: P & T.

\[ V_2 \quad 8.6170E-02 \quad \text{m}^3/\text{kg} \quad V_3 \quad 2.6146E-02 \quad \text{m}^3/\text{kg} \]
\[ H_2 \quad 400.11 \quad \text{kJ/kg} \quad H_3 \quad 443.28 \quad \text{kJ/kg} \]
\[ S_2 \quad 1.7475 \quad \text{kJ/kg-K} \quad S_3 \quad 1.7932 \quad \text{kJ/kg-K} \]

We can use the specific volume at state 2 to determine the mass flow rate since:

\[ \dot{m} = \frac{\dot{V}_2}{V_2} \]  

**Eqn 3**

Plugging values into **Eqn 3** yields:

\[ \dot{m} = 6.963 \text{ kg/min} \quad \dot{m} = 0.1160 \text{ kg/s} \]

Now, we can plug values into **Eqn 2** to complete part (a).

\[ W_{sh,comp,act} = -5.010 \text{ kW} \]
In order to determine $Q_{34}$, we must apply the steady-state form of the 1st Law for open systems to the condenser.

\[
\dot{Q}_{34} - \dot{W}_{sh,34} = \dot{m} \left[ \Delta H + \Delta E_{\text{kin}} + \Delta E_{\text{pot}} \right]_{34} \tag{Eqn 4}
\]

The shaft work for a HEX is zero because there are no moving parts. Also, since we have no information relating to either elevation or fluid velocities at any point in the cycle, we must assume that changes in kinetic and potential energies are negligible. These assumptions allow us to simplify Eqn 4 as follows.

\[
\dot{Q}_{34} = \dot{m} \left( \dot{H}_4 - \dot{H}_3 \right) \tag{Eqn 5}
\]

We can immediately lookup the properties of stream 4 because we know the values of two intensive variables: $P_4$ & $T_4$.

\[
\begin{align*}
V_4 & = 8.5811 \times 10^{-4} \text{ m}^3/\text{kg} \\
H_4 & = 249.78 \text{ kJ/kg} \\
S_4 & = 1.1695 \text{ kJ/kg-K}
\end{align*}
\]

Plugging values into Eqn 5 yields:

\[
Q_{34} = -Q_H = -22.46 \text{ kW}
\]

**Part c.)** We can determine $\text{COP}_{\text{hp}}$ from its definition.

\[
\text{COP}_{\text{hp}} = \frac{\dot{Q}_H}{\dot{W}_{\text{cycle}}} = \frac{-\dot{W}_{\text{sh,comp}}}{\dot{W}_{\text{cycle}}}
\]

We calculated $W_{\text{sh,comp}}$ in part (a) and $Q_H$ in part (b), so we can immediately plug values into Eqn 6 and complete part (c).

\[
\text{COP}_{\text{hp}} = \frac{-22.46 \text{ kW}}{-5.01 \text{ kW}} = 4.482
\]

**Part d.)** In order to determine the isentropic efficiency of the compressor, we must start from the definition of isentropic compressor efficiency.

\[
\eta_{S,\text{comp}} = \frac{\dot{H}_2 - \dot{H}_{3S}}{\dot{H}_2 - \dot{H}_3}
\]

We need to determine $H_{3S}$, the enthalpy of the effluent streams from the hypothetical isentropic compressor, in order to use Eqn 7.

The key here is that $S_{3S} = S_2$ & $P_{3S} = P_3$. Therefore, we know the values of two intensive variables at state $3S$ and we can use the NIST Webbook for R-134a to determine the value of $H_{3S}$.

\[
\begin{align*}
S_{3S} & = S_2 = 1.7475 \text{ kJ/kg-K} \\
T_{3S} & = 45.68 \degree C \\
V_{3S} & = 0.024107 \text{ m}^3/\text{kg} \\
H_{3S} & = 428.38 \text{ kJ/kg}
\end{align*}
\]

Now, we can plug values into Eqn 7 to evaluate $\eta_{S,\text{comp}}$.

\[
\begin{align*}
W_{\text{sh,pump,isen}} & = -3.28 \text{ kW} \\
W_{\text{sh,pump,act}} & = -5.01 \text{ kW}
\end{align*}
\]

\[
\eta_{S,\text{comp}} = 65.48\%
\]

**Verify:** None of the assumptions made in this problem solution can be verified.

**Answers:**

\[
\begin{align*}
a.) & \quad W_{\text{sh,comp,act}} = -5.01 \text{ kW} \\
b.) & \quad Q_{34} = -Q_H = -22.5 \text{ kW} \\
c.) & \quad \text{COP}_{\text{hp}} = 4.48 \\
d.) & \quad \eta_{S,\text{comp}} = 65.5\%
\end{align*}
\]