Problem : 6.79 - Effect of Source and Sink Temperatures on HE Efficiency - 6 pts

A heat engine operates between a source at T_H and a sink at T_C. Heat is supplied to the heat engine at a steady rate of **65,000 kJ/min**. Study the effects of T_H and T_C on the maximum power produced and the maximum cycle efficiency. For T_C = **0, 25** and **50°C**, let T_H vary from **300°C** to **1000°C**. Create plots of W_{cycle} and η_{th} as functions of T_H. Discuss the results.

Read : A Carnot Heat Engine produces the maximum power for a given heat input and also has the maximum thermal efficiency. So, this problem is really all about the Carnot HE Efficiency. We are given both reservoir temperatures and asked to construct plots of the power output and thermal efficiency as functions of reservoir temperatures. This will only require that we apply the 1st Law and the equation for the efficiency of a Carnot HE. No problem.

Given :

Q_H = **65000** kJ/min

a.)	T_C	**0**	°C
	T_H	**{300…1000}**	°C
b.)	T_C	**25**	°C
	T_H	**{0…50}**	°C
c.)	T_C	**50**	°C
	T_H	**{0…50}**	°C

Find :

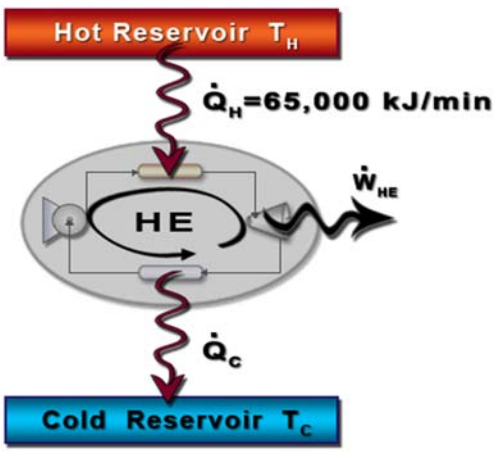
a.) Construct plots of W_{cycle} and η_{th} as a functions of T_H.

b.) Construct plots of W_{cycle} and η_{th} as a functions of T_H.

c.) Construct plots of W_{cycle} and η_{th} as a functions of T_H.

Assumptions : - The heat source and heat sink behave as true thermal reservoirs. Their temperatures remain constant regardless of how much is transferred into or out of them.

Diagram :



Equations / Data / Solve :

The equation for the thermal efficiency of a Carnot Cycle is :

$$\eta_{th,Carnot} = 1 - \frac{T_C}{T_H} \qquad \text{Eqn 1}$$

A Carnot Cycle yields the maximum thermal efficiency for a HE.

The maximum thermal efficiency also produces the maximum power. We can calculate this using the definition of the thermal efficiency of a HE and applying it to a Carnot HE.

$$\eta_{th} = \frac{\dot{W}_{cycle}}{\dot{Q}_H} \qquad \text{Eqn 2}$$

$$\eta_{th,Carnot} = \frac{\dot{W}_{max}}{\dot{Q}_H} \qquad \text{Eqn 3}$$

Solving for the maximum power output yields :

$$\dot{W}_{max} = \eta_{th,Carnot}\,\dot{Q}_H \qquad \text{Eqn 4}$$

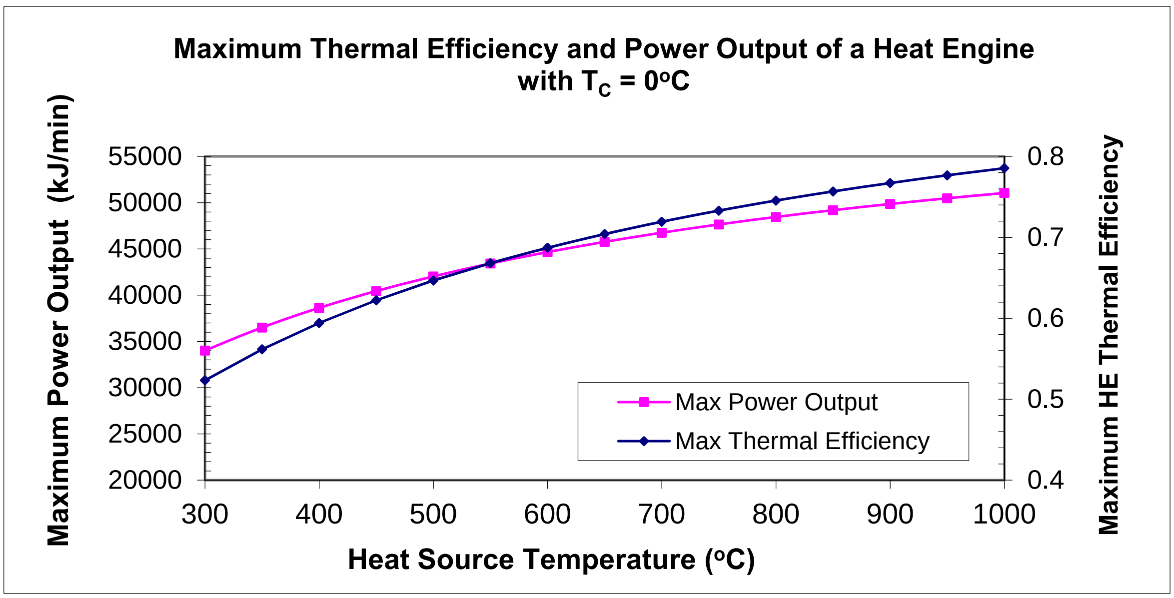
Now, we have all the equations we need to construct the plots for parts **(a), (b)** and **(c)**.

Part a.)			Part b.)			Part c.)		
T_C	0	°C	T_C	25	°C	T_C	50	°C
T_H (°C)	$\eta_{th,max}$ (kJ/min)	W_{max} (kJ/min)	T_H (°C)	$\eta_{th,max}$ (kJ/min)	W_{max} (kJ/min)	T_H (°C)	$\eta_{th,max}$ (kJ/min)	W_{max} (kJ/min)
300	0.523	34023	300	0.480	31187	300	0.436	28352
350	0.562	36508	350	0.522	33900	350	0.481	31293
400	0.594	38624	400	0.557	36210	400	0.520	33796
450	0.622	40448	450	0.588	38201	450	0.553	35954
500	0.647	42036	500	0.614	39934	500	0.582	37832
550	0.668	43431	550	0.638	41457	550	0.607	39482
600	0.687	44666	600	0.659	42805	600	0.630	40944
650	0.704	45767	650	0.677	44007	650	0.650	42247
700	0.719	46755	700	0.694	45086	700	0.668	43416
750	0.733	47647	750	0.709	46059	750	0.684	44471
800	0.745	48455	800	0.722	46941	800	0.699	45427
850	0.757	49192	850	0.735	47745	850	0.712	46298
900	0.767	49866	900	0.746	48481	900	0.725	47095
950	0.777	50484	950	0.756	49156	950	0.736	47827
1000	0.785	51054	1000	0.766	49778	1000	0.746	48502

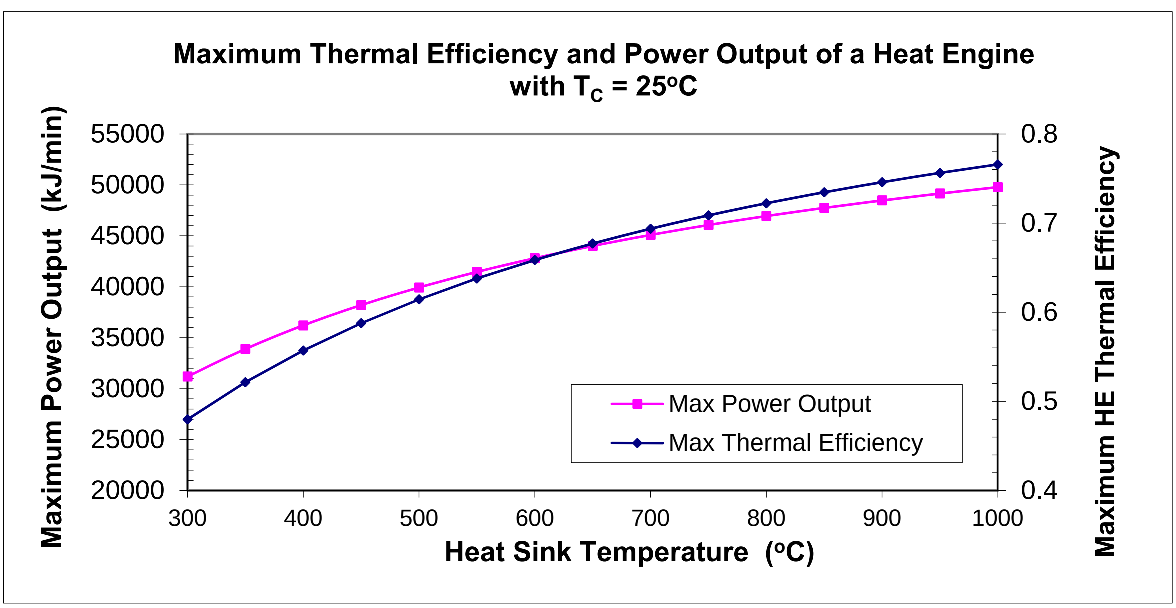
Verify : None of the assumptions made in this problem solution can be verified.

Answers :

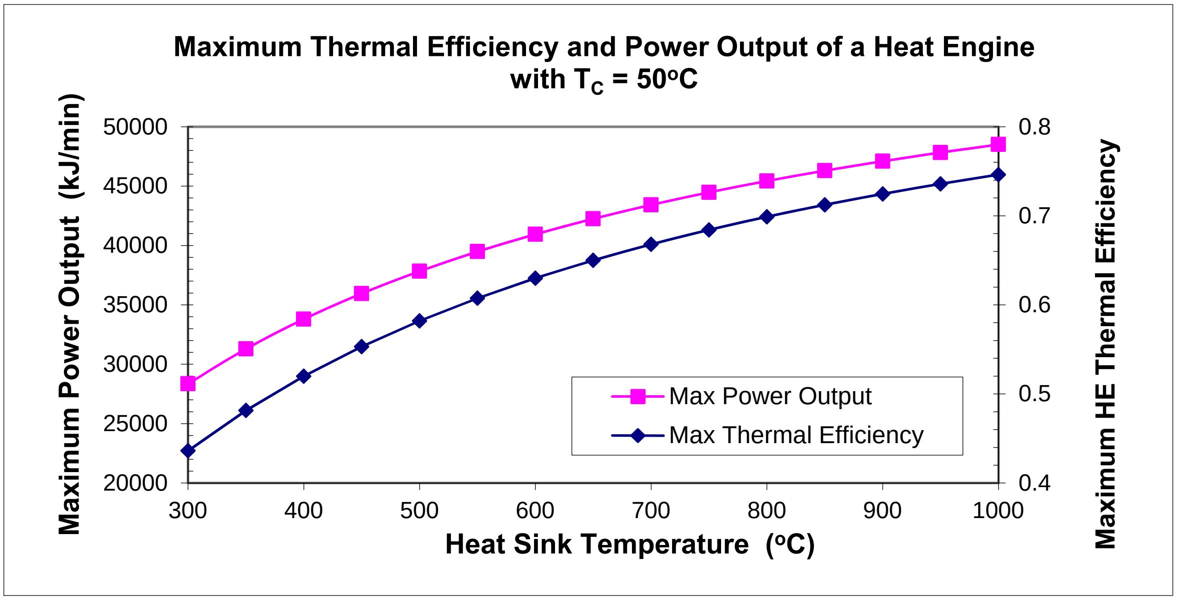
Part (a)



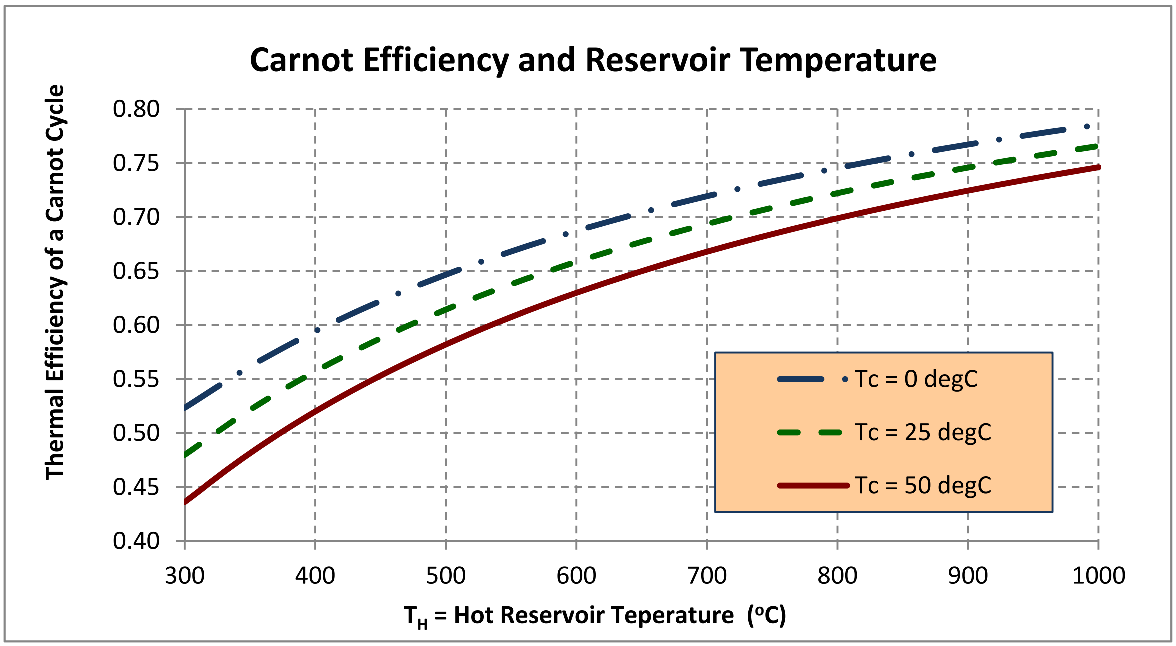
Part (b)

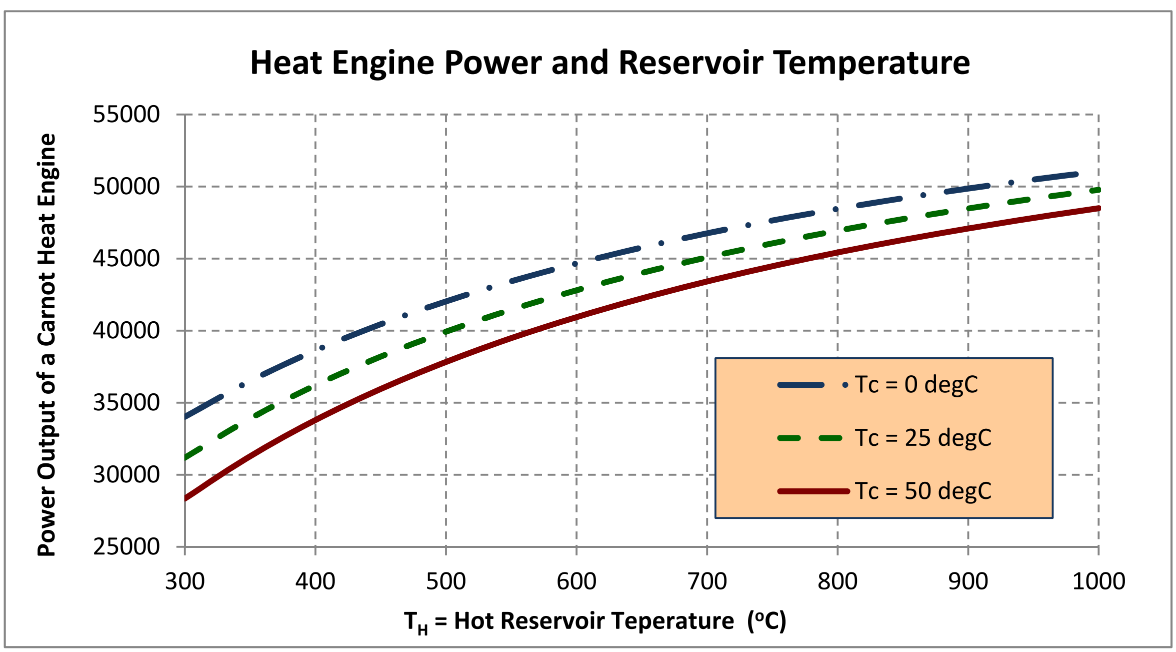


Part (c)



Other Acceptable Plots :





Problem : 6.85 - Thermal Efficiency of a Geothermal Power Plant - 3 pts

A geothermal power plant uses geothermal water extracted at **150°C** at a rate of **210 kg/s** as the heat source and produces **8000 kW** of net power. The geothermal water leaves the plant at **90°C**. If the enviromnment temperature is **25°C**, determine …

a.) The actual thermal efficiency.

b.) The maximum possible thermal efficiency.

c.) The actual rate of heat rejection from this power plant.

Read : The key to this problem is using the change in the temperature of the geothermal water to evaluate $\mathbf{Q_H}$.
With some reliable assumtions we can easily determine $\mathbf{Q_H}$ from the heat capacity of the geothermal water.
Then, we can use the definition of thermal efficiency to complete **part (a)** and the 1st Carnot Principle to complete **part (b)**.
We can then apply the 1st Law to the entire power cylce to determine $\mathbf{Q_C}$ and complete **part (c)**.

Given :

m_{geo}	**210**	kg/s	W_s	**8000**	kW
$T_{geo,in}$	**150**	°C	$T_{geo,out}$	**90**	°C
	423.15	K	T_{surr}	**25**	°C
				298.15	K

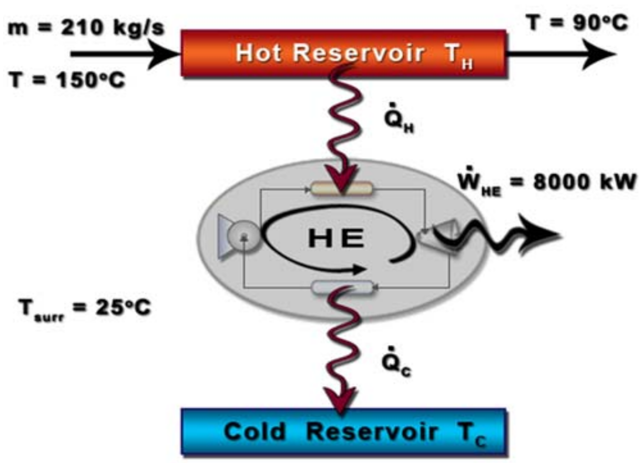
Find :

a.)	η	???	%				
b.)	η_{max}	???	%	**c.)**	Q_C	???	kW

Assumptions :

1 - All of the heat given up by the geothermal water is taken in by the heat engine.
2 - Changes in kinetic and potential energy of the geothermal water are negligible.
3 - The geothermal water is an incompressible liquid over the range of pressure from its inlet to its outlet condition.
4 - The heat capacity of the geothermal water is the same as the heat capacity of water and that this heat capacity is constant.

Diagram :



Solution :

Part a.) We can determine the actual thermal efficiency of the power plant directly from its definition.

$$\eta = \frac{\textbf{Desired}}{\textbf{Required}} = \frac{\dot{W}_{cycle}}{\dot{Q}_H} \qquad \textbf{Eqn 1}$$

Since $\mathbf{W_S}$ is given, we need to determine $\mathbf{Q_H}$ before we can use **Eqn 1** to complete this part of the problem.
$\mathbf{Q_H}$ is the amount of energy removed from the geothermal water in the plant. So, we can apply the 1st Law to a system consisting of just the geothermal water.

$$Q_{water} - \cancel{W}_{S,water} = \dot{m}\left(\Delta\hat{H} + \cancel{\Delta\hat{E}}_{kin} + \cancel{\Delta\hat{E}}_{pot}\right) \qquad \textbf{Eqn 2}$$

We can simplify **Eqn 2** if we assume that changes in kinetic and potential energies are negligible and that any shaft work produced or consumed are taken into account in the $\mathbf{W_S}$ value given in the problem statement.

$$Q_{water} = \dot{m}\,\Delta\hat{H} \qquad \textbf{Eqn 3}$$

Next, we must recognize the heat leaving the geothermal water is entering the power cycle. In terms of the sign convention for heat transfer, Q_H is equal in magnitude to Q_{water}, but opposite in sign.

$$Q_H = -Q_{water} \qquad \text{Eqn 4}$$

If we further assume that the geothermal water is an incompressible liquid over the range of pressure from its inlet to its outlet condition and that the heat capacity of the geothermal water is the same as the heat capacity of water and that this heat capacity is constant, then :

$$Q_H = -Q_{water} = \dot{m}_{geo} \int_{T_{geo,in}}^{T_{geo,out}} \hat{C}_{P,w}\, dT = \dot{m}_{geo}\left(T_{geo,out} - T_{geo,in}\right) \qquad \text{Eqn 5}$$

Where : $C_{P,w}$ 4.22 kJ/kg-K

Plugging values into **Eqn 5** yields : Q_H 53172 kW

Plugging values back into **Eqn 1** yields : η **15.05** **%**

Part b.) The 1st Carnot Principle tells us that a reversible Carnot Cycle has the maximum efficiency of any cycle operating between the same two thermal reservoirs. The Carnot Efficiency for a power cycle is given by:

$$\eta_{th,Carnot} = \frac{\dot{W}_{max}}{\dot{Q}_H} = 1 - \frac{T_C}{T_H} \qquad \text{Eqn 6}$$

In this case, the maximum efficiency would be achieved if the geothermal water behaved a true thermal reservoir and could supply heat while remaining constantly at $T_{geo,in}$ = **150°C** instead of dropping to $T_{geo,out}$ = **90°C**. So, for the maimum possible thermal efficiency, we will use T_H = **150°C** = **423.15 K**. The cold reservoir is the surroundings at T_C = **25°C** = **298.15 K**.

T_H 423.15 K T_C 298.15 K

Now, we can plug these values into **Eqn 6**. η_{max} **29.54** **%**

Part c.) We can determine Q_C by applying the 1st Law to the entire power cycle. The result, with no sign convention (all quantities are positive), is:

$$\dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_C \qquad \text{Eqn 7}$$

Solving **Eqn 7** for Q_C yields :

$$\dot{Q}_C = \dot{Q}_H - \dot{W}_{cycle} \qquad \text{Eqn 8}$$

Plugging values into **Eqn 8** yields : Q_C **45172** **kW**

Verify : None of the assumptions made in this problem solution can be verified.

Answers :

a.) η **15.0** **%**

b.) η_{max} **29.5** **%**

c.) Q_C **45200** **kW**

Problem : **6.107 - Carnot HE Used to Drive a Carnot Refrigerator - 6 pts** **5-May-11**

A Carnot Heat Engine receives heat from a reservoir at **900°C** at a rate of **800 kJ/min** and rejects the waste heat to the ambient air at **27°C**. The entire work output of the heat engine is used to drive a refrigerator that removes heat from the refrigerated space at **-5°C** and rejects heat to the same ambient air at **27°C**. Determine:

a.) The maximum rate of heat removal from the refrigerated space.
b.) The total rate of heat rejection to the ambient air.

Read : The first key to this problem is that the maximum amount of heat will be removed from the refrigerated space when BOTH the **HE** and the **Ref** operate on reversible thermodynamic cycles.

The second key is that all of the work produced by the HE is used to drive the Ref, so $\mathbf{W_{HE} = W_{Ref} = W_{cycle}}$.

Once these key aspects of the problem are recognized the solution consists of the algebraic manipulation of the 1st Law for each cycle and the equations for $\mathbf{COP_R}$ and η_{th} for reversible cycles in terms of the reservoir temperatures.

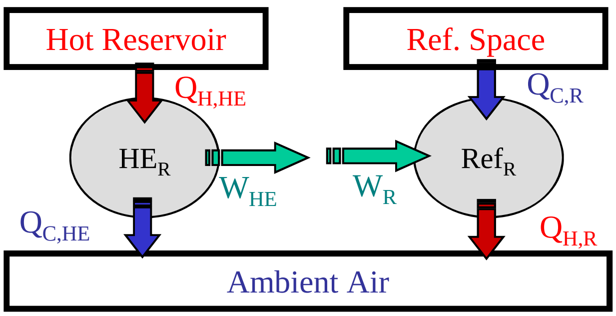
Given :

$T_{H,HE}$	**900**	°C	$Q_{H,HE}$	**800**	kJ/min
	1173.15	K	$T_{H,Ref}$	300.15	K
$T_{C,HE}$	**27**	°C	$T_{C,Ref}$	**-5**	°C
	300.15	K		268.15	K

Find :

a.)	Maximum	$Q_{C,R}$	???	kJ/min
b.)	$Q_{C,HE} + Q_{H,R}$		???	kJ/min

Diagram :



Assumptions :

1 - The **HE** and **Ref** are reversible thermodynamic cycles operating at steady-state.
2 - The cycles exchange heat with true thermal reservoirs whose temperatures do not change during this process.
3 - All of the work produced by the **HE** is used to drive the **Ref**.

Equations / Data / Solve :

Part a.) The maximum value of $\mathbf{Q_{C,R}}$ will be achieved when BOTH the **HE** and the **Ref** operate on reversible cycles. So, we assume that both cycles are reversible.

We can determine $\mathbf{Q_{C,R}}$ from the $\mathbf{COP_R}$:

$$COP_R = \frac{Q_{C,R}}{W_R} \qquad \textbf{Eqn 1}$$

Solving for $\mathbf{Q_{C,R}}$ gives us :

$$Q_{C,R} = W_R \cdot COP_R \qquad \textbf{Eqn 2}$$

So, we need to evaluate $\mathbf{COP_R}$ and $\mathbf{W_R}$ so we can use **Eqn 2** to evaluate $\mathbf{Q_{C,R}}$.
Because all of the work produced by the **HE** is used to drive the refrigerator :

$$W_R = W_{HE} = W_{cycle} \qquad \textbf{Eqn 3}$$

We can use the thermal efficiency of the **HE** to determine $\mathbf{W_{cycle}}$. Since we know the temperature of all four thermal reservoirs, we can determine the thermal efficiency of the **HE** and the $\mathbf{COP_R}$.

$$\eta_{th,rev} = 1 - \frac{T_C}{T_H} = \frac{W_{cycle}}{Q_{H,HE}} \qquad \textbf{Eqn 4}$$

$$COP_{R,rev} = \frac{1}{\frac{T_H}{T_C} - 1} \qquad \textbf{Eqn 5}$$

Plugging in values gives us :

$\eta_{th,rev}$	0.7442
$COP_{R,rev}$	8.380

Next, we can solve **Eqn 4** for $\mathbf{W_{cycle}}$:

$$W_{cycle} = \eta_{th,rev} \cdot Q_{H,HE} \qquad \textbf{Eqn 6}$$

W_{cycle}	595.3	kJ/min

Finally, we can plug values back into **Eqn 2** (where $\mathbf{W_R = W_{cycle}}$).

$Q_{C,R}$	4989	kJ/min

Part b.) We can solve this part of the problem by applying the 1st Law to each cycle.

$$Q_{H,R} = Q_{C,R} + W_{cycle} \qquad \textbf{Eqn 7}$$

$$Q_{H,HE} = Q_{C,HE} + W_{cycle} \qquad \textbf{Eqn 8}$$

Solve **Eqn 8** for $\mathbf{Q_{C,HE}}$:

$$Q_{C,HE} = Q_{H,HE} - W_{cycle} \qquad \textbf{Eqn 9}$$

Now, we can plug values into **Eqns 7 & 9** :

$Q_{H,R}$	5583.9	kJ/min
$Q_{C,HE}$	204.7	kJ/min
$Q_{C,HE} + Q_{H,R}$	5788.6	kJ/min

Verify : None of the assumptions made in this problem solution can be verified.

Answers :

a.)	$\mathbf{Q_{C,R}}$	**4989**	**kJ/min**
b.)	$\mathbf{Q_{C,HE} + Q_{H,R}}$	**5789**	**kJ/min**

Problem : 6.110 - Actual and Maximum COP of an Air-Conditioner - 8 pts 5-May-11

An air-conditioner with **R-134a** as the working fluid is used to keep a room at **23°C** by rejecting the waste heat to the outside air at **37°C**. The room gains heat through the walls and the windows at a rate of **250 kJ/min** while the heat generated by the computer, TV, and lights amounts to **900 W**. The refrigerant enters the compressor at **400 kPa** as a saturated vapor at a rate of **100 L/min** and leaves at **1200 kPa** and **70°C**. Deterine...

a.) The actual **COP**
b.) The maximum **COP**
c.) The minimum volumetric flow rate of the **R-134a** at the compressor inlet for the same compressor inlet and exit conditions.

Read : It s important to recognize that when the room temperature is steady, $\mathbf{Q_C}$ must be equal to the rate at which heat flows into the room from all sources. In this case, $\mathbf{Q_C}$ must be equal to the sum of the rate at which heat flows into the room from the hot outdoor air and the rate at which electrical appliances dissipate energy in the form of heat inside the room.

Part (a) is an application of the 1st Law and the definition of COP for a refrigeration cycle.
Part (b) is an application of the definition of COP for a refrigeration cycle and the Kelvin Principle.
Part (c) is an application of the 1st Law to the reversible refrigeration cycle from **part (b)**. The key is to recognize that a reversible refrigeration cycle requires the minimum work to accomplish a given refrigeration process.

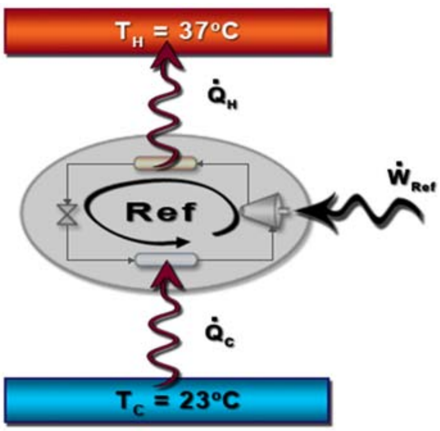
Given :

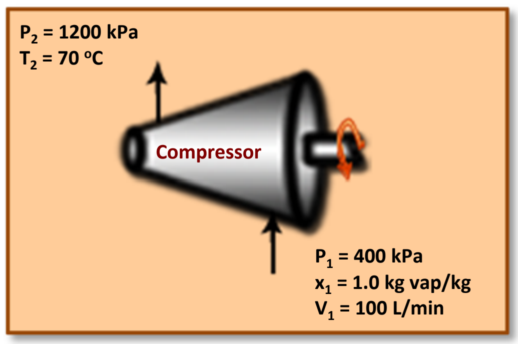
Chemical :	R-134a			P_1	400	kPa
Units :	SI_C			x_1	1	kg vap/kg
T_{room}	23	°C		V_1	100	L/min
	296.15				0.001667	m^3/s
$T_{outside}$	37	°C				
	310.15			P_2	1200	kPa
Q_{walls}	250	kJ/min		T_2	70	°C
	4167	W				
Q_{elec}	900	W				

Find :

a.)	COP	???	
b.)	COP_{max}	???	
c.)	$V_{1,min}$	???	m^3/s

Diagram :





Assumptions :

1 - The air-conditioner operates at steady-state.
2 - The compressor is the only process in the air-conditioner cycle that produces or consumes work or power.
3 - The compressor is adiabatic.
4 - Changes in kinetic and potential energies are negligible in the compressor.

Equations / Data / Solve :

	Compressor Inlet 1	Compressor Outlet 2	
T	8.9	70	°C
P	400	1200	kPa
T_{sat}	8.9	46.3	°C
x	1	N/A	kg vap/kg
H	403.72	448.76	kJ/kg
V	0.051207	0.019502	m^3/kg
Phase	Saturated Vapor	Superheated Vapor	

Part a.) Since we need to determine the actual **COP**, let's begin with its definition:

$$COP_{ref} = \frac{\dot{Q}_C}{\dot{W}_{ref}} \quad \textbf{Eqn 1}$$

In **Eqn 1**, $\mathbf{Q_C}$ is the heat removed from the room or or interior space by the air-conditioner. When the room temperature is steady, $\mathbf{Q_C}$ must be equal to the rate at which heat flows into the room from all sources. In this case, $\mathbf{Q_C}$ must be equal to the sum of the rate at which heat flows into the room from the hot outdoor air, $\mathbf{Q_{walls}}$, and the rate at which electrical appliances dissipate energy in the form of heat inside the room, $\mathbf{Q_{elec}}$.

A mathematical expression of this idea is :

$$\dot{Q}_C = Q_{walls} + Q_{elec} \quad \textbf{Eqn 2}$$

Q_C 5.067 kW

Next, we need to determine the net rate at which work or power is supplied to the cycle. Here we assume that the compressor is the only unit or process where work enters or leaves the cycle. There is no turbine. An expansion valve or throttling device is used instead to reduce the pressure from the high pressure at the condenser outlet to the low pressure at the evaporator inlet.

We can apply the 1st Law to determine the shaft work input at the compressor. We have to be careful because $\mathbf{W_S}$ in the 1st Law will have a negative value because wor is input to the compressor from the surroundings. But when we shift gears and plug values into **Eqn 1** for $\mathbf{W_{ref}}$ and $\mathbf{Q_C}$, all values must be positive because **Eqn 1** was determined based on a tie-fighter diagram in which a sign convention is not used. As a result, $\mathbf{W_{ref} = -W_{S,comp}}$.

$$\cancel{\dot{Q}_{comp}} - \dot{W}_{S,comp} = \dot{m}\left(\Delta\hat{H} + \cancel{\Delta\hat{E}_{kin}} + \cancel{\Delta\hat{E}_{pot}}\right) \quad \textbf{Eqn 3}$$

We can simplify **Eqn 3** by assuming that the compressor is adiabatic and that changes in kinetic and potential energies are negligible. The result is :

$$\dot{W}_{S,comp} = \dot{m}\left(\hat{H}_1 - \hat{H}_2\right) < 0 \quad \textbf{Eqn 4}$$

The volumetric flow rate of R-134a at the compressor inlet was given in the problem statement. We can use this value to determine the mass flow rate of R134a through the compressor using :

$$\dot{m} = \frac{\dot{V}_1}{\hat{V}_1} \quad \textbf{Eqn 5}$$

In order to use **Eqn 5**, we need to determine the specific volume of the R-134a at the compressor inlet. Fortunately, we know that the R-134a is a <u>satuarted vapor</u> at **400 kPa** at the compressor inlet. We can look-up the specific volume and any other intensive property, including the specific enthalpy, in **thermodynamic tables**, in the **NIST Webbook** or using the **TFT plug-in**. I chose to use the **TFT plug-in**.

V_1 0.051207 m^3/kg H_1 403.72 kJ/kg

Plugging values into **Eqn 5** yields : m 0.03255 kg/s

Now, we need to determine $\mathbf{H_2}$ so we can use **Eqn 4**. Both $\mathbf{T_2}$ and $\mathbf{P_2}$ were given in the problem statement, so we can look-up the specific enthalpy, in **thermodynamic tables**, in the **NIST Webbook** or using the **TFT plug-in**. Since I chose to use the **TFT plug-in** before, I will use it again to avoid any reference state inconsistencies.

H_2 448.76 kJ/kg

Now, we can plug values into **Eqn 4** to evaluate $\mathbf{W_{S,comp}}$. We can also evaluate $\mathbf{W_{ref}}$ since $\mathbf{W_{ref} = -W_{S,comp}}$.

$\mathbf{W_{S,comp}}$ -1.4660 kW $\mathbf{W_{ref}}$ 1.4660 kW

Now, we can go back to **Eqn 1** to evaluate the **COP** and complete this part of the problem.

$\boxed{\textbf{COP} \quad \textbf{3.456}}$

Part b.) The 1st Carnot Principle tells us that a Carnot Cycle will yield the maximum efficiency of any cycle operating between the same two thermal reservoirs. We can evaluate the **COP** of a Carnot refrigeration (air-conditioning) cycle from the temperatures of the two thermal reservoirs with which the cycle interacts using :

$$COP_{R,rev} = \frac{1}{\frac{T_H}{T_C} - 1} \quad \textbf{Eqn 6}$$

The cold reservoir is the air inside the room being air-conditioned, so **T_C = 23°C = 296.15K**.
The hot reservoir is the outside air, so **T_H = 37°C = 310.15K**.
Plugging these temperatures into **Eqn 6** yields :

COP_{max}	**21.15**

Part c.) The most efficient air-conditioner will have the largest **COP**. This is the **COP** we calculated in **part (b)**. This air-conditioner requires the smallest $\mathbf{W_{ref}}$ to accomplish the given refrigeration (air-conditioning) task. So, in this part of the problem, we need to determine the necessary refrigerant volumetric flow rate at the compressor inlet that corresponds to $\mathbf{COP_{max}}$ from **part (b)**. In equation form :

$$COP_{max} = \frac{\dot{Q}_C}{\dot{W}_{ref,min}} \quad \textbf{Eqn 7}$$

Or :

$$\dot{W}_{ref,min} = \frac{\dot{Q}_C}{COP_{max}} \quad \textbf{Eqn 8}$$

But, from **part (a)** we also know that :

$$\dot{W}_{ref,min} = -\dot{W}_{S,comp} = \dot{m}\left(\hat{H}_2 - \hat{H}_1\right) > 0 \quad \textbf{Eqn 9}$$

The relationship between the mass flow rate and the volumetric flow rate can be obtained by rearranging **Eqn 5**, as follows.

$$\dot{V}_1 = \dot{m}\ \hat{V}_1 \quad \textbf{Eqn 10}$$

We can determine the mass flow rate of the R-134a by solving **Eqn 9** for **m** :

$$\dot{m} = \frac{\dot{W}_{ref,min}}{\hat{H}_2 - \hat{H}_1} \quad \textbf{Eqn 11}$$

Since the state of the R-134a at the inlet and outlet are the same in our, ideal, maximum **COP**, Carnot air-conditioner as in the real air-conditioner, we can go ahead and plug values into **Eqns 8, 11 & 10** to complete this problem.

$W_{ref,min}$	0.2395	kW	$V_{1,min}$	0.0002723	m^3/s
m	0.00532	kg/s	**$V_{1,min}$**	**16.339**	**L/min**

Verify : None of the assumptions made in this problem solution can be verified.

Answers :

a.)	**COP**	**3.46**	c.)	**$V_{1,min}$**	**16.3 L/min**
b.)	**COP_{max}**	**21.2**			

Problem : 6.134 - Thermal Efficiency of Heat Engines in Series - 4 pts

Consider two Carnot heat engines operating in series. The first engine receives heat from a reservoir at **1800 K** and rejects waste heat to another reservoir at temperature **T**. From the reservoir at temperature **T**, the second heat engine receives the heat energy rejected by the first heat engine, converts some of it to work, and rejects the rest to a third thermal reservoir at **300 K**. If the thermal efficiencies of the heat engines are the same, determine the temperature, **T**.

Read : The key to this problem is that both heat engines are Carnot heat engines.
The efficiency of a Carnot heat engine depends only on the temperatures of the reservoirs with which it interacts.
Because the problem statement tells us that the efficiencies of the two heat engines are the same, we can determine the temperature of the unknown thermal reservoir, $\mathbf{T_{res}}$.

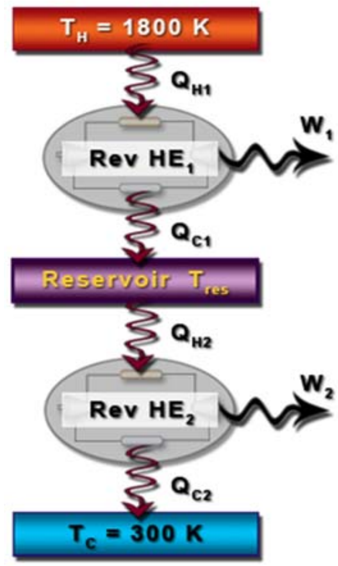
Given :

T_H	1800	K
T_C	300	K

Find :

T_{res}	???	K

Diagram :



Assumptions :

1 - Both engines are Carnot heat engines operating at steady state.
2 - All three reservoirs are true thermal reservoirs and so their temperatures do not change.

Solution : The key to this problem seems to be that thermal efficiencies of the two Carnot Heat Engines are the same. So, let's start from the definition of thermal efficiency.

$$\eta = \frac{\text{Desired}}{\text{Required}} = \frac{W_{cycle}}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H} \qquad \textbf{Eqn 1}$$

Next, we can apply the Kelvin Principle to express the thermal efficiency in **Eqn 1** in terms of the temperatures of the reservoirs that the Carnot Heat Engine interacts with.

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} \qquad \textbf{Eqn 2}$$

$$\eta = 1 - \frac{T_C}{T_H} \qquad \textbf{Eqn 3}$$

Now, we can apply **Eqn 3** to both of the heat engines in our system.

$$\eta_1 = 1 - \frac{T_{res}}{T_H} \qquad \textbf{Eqn 4}$$

$$\eta_2 = 1 - \frac{T_C}{T_{res}} \qquad \textbf{Eqn 5}$$

Since the efficiencies of the two heat engines are equal, we can combine **Eqns 4 & 5** to get :

$$\eta_1 = 1 - \frac{T_{res}}{T_H} = 1 - \frac{T_C}{T_{res}} = \eta_2 \qquad \textbf{Eqn 6}$$

The **1**s cancel in **Eqn 6**, leaving us with :

$$\frac{T_{res}}{T_H} = \frac{T_C}{T_{res}} \qquad \textbf{Eqn 7}$$

Now, we can solve **Eqn 7** for the unknown $\mathbf{T_{res}}$, as follows.

$$T_{res}^2 = T_H\, T_C \qquad \textbf{Eqn 8}$$

$$T_{res} = \sqrt{T_H\, T_C} \qquad \textbf{Eqn 9}$$

Now, we can plug values into **Eqn 9** to complete our solution :

T_{res}	734.85	K

Verify : None of the assumptions made in this problem solution can be verified.

Answers :

T_{res}	735	K

Problem : 1 - "Show That" Using the K-P Statement of the 2nd Law - 6 pts 5-May-11

Using the Kelvin-Planck statement of the 2nd Law, demonstrate the following corollaries.

a.) The coefficient of performance (**COP**) of an irreversible heat pump cycle is always less than the **COP** of a reversible heat pump when both heat pumps exchange heat with the same two thermal reservoirs.

b.) All reversible heat pump cycles exchanging heat with the same two thermal reservoirs have the same **COP**.

Read : The solution to this "show that" problem requires the clever use of tie-fighter diagrams to show that the situations described in the problem statement consitute violations of the Kelvin-Planck statement of the 2nd Law.

The key to solving the problem is to keep in mind that reversible heat pumps can be reversed. When reversed, a reversible heat pump becomes a reversible heat engine.

Given : None.

Find :
a.) Show that $\mathbf{COP_{HP,Rev} > COP_{HP,IRR}}$ for heat pumps operating between the same to thermal reservoirs.
b.) Show that $\mathbf{COP_{HP}}$ is the same for all reversible heat pumps operating between the same to thermal reservoirs.

Solution :

Part a.) Consider the two heat pumps shown in the diagram, below. The heat pump on the left is reversible and the heat pump on the right is irreversible.

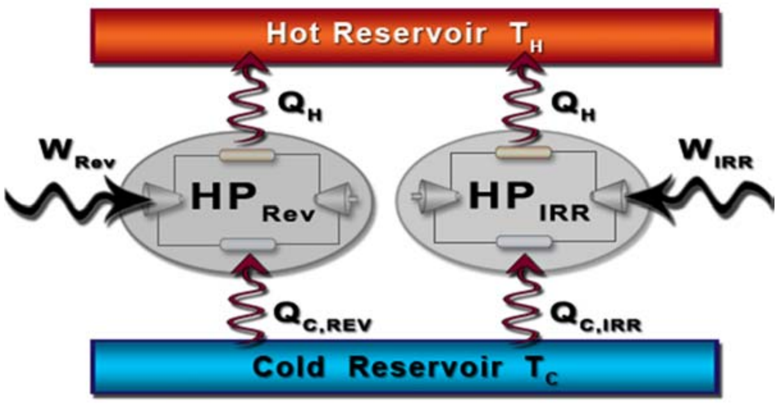


Figure 1

Let's assume that the irreversible heat pump is more efficient, and therefore has a higher coefficient of performance, than the reversible heat pump.

$$\mathbf{COP_{HP,IRR} > COP_{HP,Rev}} \quad \text{Eqn 1}$$

The definition of **COP** for a heat pump is :

$$\mathbf{COP_{HP} = \frac{Q_H}{W_{cycle}}} \quad \text{Eqn 2}$$

Combining **Eqns 1 & 2** yields :

$$\mathbf{\left(\frac{Q_H}{W_{cycle}}\right)_{IRR} > \left(\frac{Q_H}{W_{cycle}}\right)_{Rev}} \quad \text{Eqn 3}$$

In order to make a consistent and fair comparison of the two heat pumps, we can require each of them to deliver the same amount of heat to the hot reservoir. So, $\mathbf{Q_{H,IRR} = Q_{H,Rev}}$. In this case, **Eqn 3** simplifies to :

$$\mathbf{W_{Rev} > W_{IRR}} \quad \text{Eqn 4}$$

We can also apply the 1st Law to either HP cycle :

$$\mathbf{Q_H = Q_C + W_{cycle}} \quad \text{Eqn 5}$$

Combining **Eqns 4 & 5** yields :

$$\mathbf{\cancel{Q_H} - Q_{C,Rev} > \cancel{Q_H} - Q_{C,IRR}} \quad \text{Eqn 6}$$

The $\mathbf{Q_H}$ terms in **Eqn 6** cancel to give us :

$$\mathbf{Q_{C,IRR} > Q_{C,Rev}} \quad \text{Eqn 7}$$

Now that we have a good understanding of the implications of the assumption that **$COP_{HP,IRR}$ > $COP_{HP,Rev}$** , we need to reverse our reversible HP. This converts it into the HE shown at left in the diagram, below.

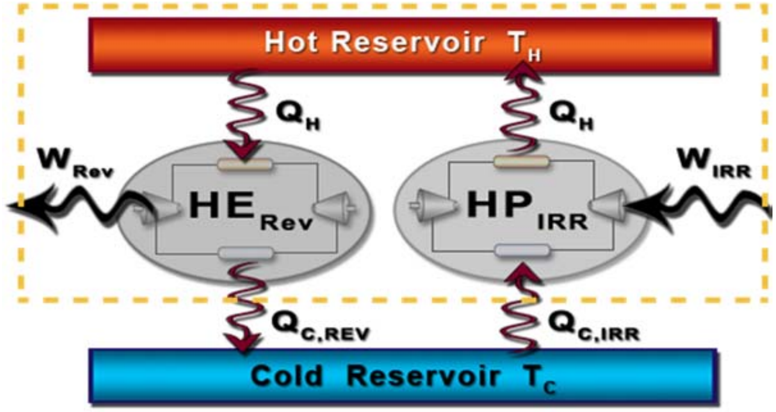


Figure 2

Because **HE_R** in **Figure 2** absorbs the same amount of heat, **Q_H**, from the hot reservoir as **HP_I** rejects to the hot reservoir, there is zero net heat transfer to or from the hot reservoir. As a result, the hot reservoir can be combined with **HE_R** and **HP_I** to form a new system. The boundary of this new system is indicated by a dashed yellow system boundary line in **Figure 2**.

This new system absorbs a net amount of heat eqal to :

$$Q_{C,IRR} - Q_{C,Rev} \qquad \textbf{Eqn 8}$$

The new system completely converts the heat it absorbs into a net amount of work equal to :

$$W_{Rev} - W_{IRR} \qquad \textbf{Eqn 9}$$

This new system violates the Kelvin-Planck Statement of the 2nd Law because it has a 100% thermal efficiency: it completely converts heat into work. This is not possible. Therefore, our assumption that **$COP_{HP,IRR}$ > $COP_{HP,Rev}$** is not possible.

This is not quite a "proof", in part because we have not considered the possibility that **$COP_{HP,IRR}$ = $COP_{HP,Rev}$**.
But intuitively, **$COP_{HP,IRR}$ = $COP_{HP,Rev}$** cannot be true for all HP cycles because it would mean that the COPHP would be a function of reservoir temperatures only and would not depend on the magnitude of the irreversibilities present. Experience has shown that greater friction and other irrevesibilities do redcue the COP of real, irreversible HPs. This is true and correct, but it is not quite a proof.

Part b.) Consider the two heat pumps shown in the diagram, below. Both heat pumps are reversible.

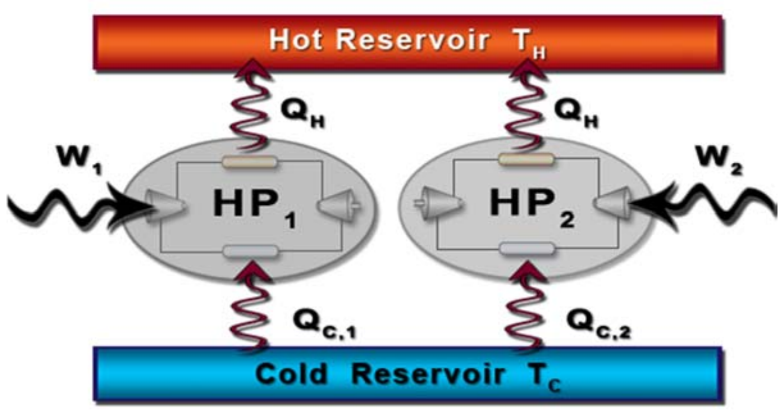


Figure 3

Let's assume that reversible heat pump 2 is more efficient, and therefore has a higher coefficient of performance, than reversible heat pump 1.

$$COP_{HP,2} > COP_{HP,1} \qquad \textbf{Eqn 10}$$

Combining **Eqns 2 & 10** yields :

$$\left(\frac{Q_H}{W_{cycle}}\right)_2 > \left(\frac{Q_H}{W_{cycle}}\right)_1 \qquad \textbf{Eqn 11}$$

In order to make a consistent and fair comparison of the two heat pumps, we can require each of them to deliver the same amount of heat to the hot reservoir. So, **$Q_{H,1}$ = $Q_{H,2}$**. In this case, **Eqn 11** simplifies to :

$$W_1 > W_2 \qquad \textbf{Eqn 12}$$

We can now use the 1st Law as applied to a HP cycle, shown in **Eqn 5**, to express **Eqn 12** in terms of heat only.

$$\cancel{Q_H} - Q_{C,1} > \cancel{Q_H} - Q_{C,2}$$ **Eqn 13**

The $\mathbf{Q_H}$ terms in **Eqn 13** cancel to give us : $$Q_{C,2} > Q_{C,1}$$ **Eqn 14**

Now, since both HPs are reversible, we can choose to reverse either one of them. It is easiest to reach the result if we reverse $\mathbf{HP_1}$. The result is shown in **Figure 4**, below.

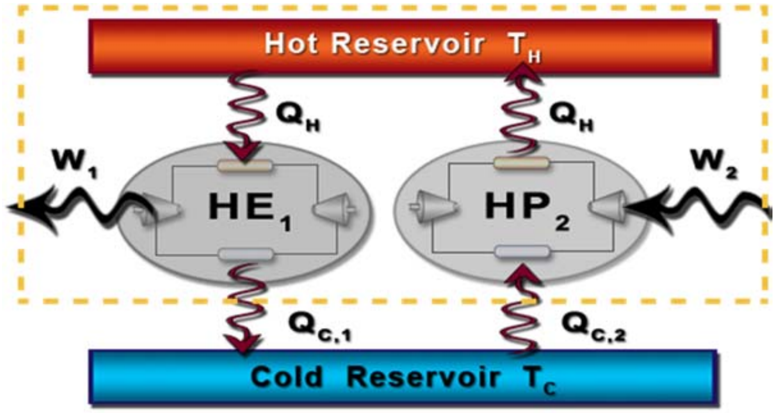


Figure 4

Because $\mathbf{HE_1}$ in **Figure 2** absorbs the same amount of heat, $\mathbf{Q_H}$, from the hot reservoir as $\mathbf{HP_2}$ rejects to the hot reservoir, there is zero net heat transfer to or from the hot reservoir. As a result, the hot reservoir can be combined with $\mathbf{HE_1}$ and $\mathbf{HP_2}$ to form a new system. The boundary of this new system is indicated by a dashed yellow system boundary line in **Figure 4**.

This new system absorbs a net amount of heat eqal to : $$Q_{C,2} - Q_{C,1}$$ **Eqn 15**

The new system completely converts the heat it absorbs into a net amount of work equal to : $$W_1 - W_2$$ **Eqn 16**

This new system violates the Kelvin-Planck Statement of the 2nd Law because it has a 100% thermal efficiency: it completely converts heat into work. This is not possible.

Therefore, our assumption that $\mathbf{COP_{HP,2} > COP_{HP,1}}$ is <u>not</u> <u>possible</u>. We conclude that all reversible heat pumps operating between the same two thermal reservoirs must have the $\mathbf{COP_{HP}}$.

Verify : No assumptions were required in the solution of this problem.

Answers : There are no "answers" as such in a show-that problem like this one.

A refrigeration cycle operating between two reservoirs receives Q_C from a cold reservoir at T_C = **250 K** and rejects Q_H to a hot reservoir at T_H = **300 K**. For each of the following cases, determine whether the cycle is reversible, irreversible or impossible.

a.)	Q_C = 1000 kJ and W_{cycle} = 400 kJ	**c.)**	Q_H = 1500 kJ and W_{cycle} = 200 kJ
b.)	Q_C = 1500 kJ and Q_H = 1800 kJ	**d.)**	COP = 4

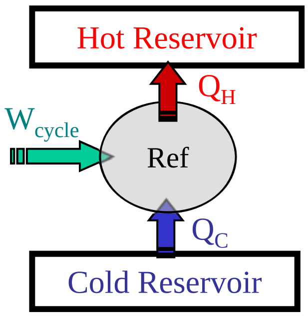
Read : The key is to apply the 1st and 2nd Carnot Principles to reversible refrigeration cycles.
This allows us to determine whether a hypothetical cycle, such as the ones given here, are reversible, irreversible or impossible by comparing their COP_R to the COP_R of a reversible refrigeration cycle operating between the same two thermal reservoirs.

Given :

	T_C	250	K	**b.)**	Q_C	1500	kJ
	T_H	300	K		Q_H	1800	kJ
a.)	Q_C	1000	kJ	**c.)**	Q_H	1500	kJ
	W_{cycle}	400	kJ		W_{cycle}	200	kJ
				d.)	COP_R	4	

Find : Reversible ?
Irreversible ?
Impossible ?

Diagram :



Assumptions :
1 - The refrigerator operates on a thermodynamic cycle.
2 - The cycle exchanges heat with true thermal reservoirs whose temperatures do not change during this process.

Equations / Data / Solve :

The keys to this problem are the two Carnot Principles and the Kelvin Principle.

1st Carnot Principle : $$COP_{R,rev} > COP_{R,irrev} \quad \text{Eqn 1}$$

2nd Carnot Principle : $$COP_{R,rev,1} > COP_{R,rev,2} \quad \text{Eqn 2}$$

Therefore, IF : $COP_R > COP_{R,rev}$ Then, the cycle is **impossible**.

$COP_R = COP_{R,rev}$ Then, the cycle is **reversible**.

$COP_R < COP_{R,rev}$ Then, the cycle is **irreversible**.

So, the key is to determine the **COP$_R$** of the actual refrigerator and compare it to the **COP$_R$** of a reversible refrigerator operating between the same two thermal reservoirs.

Let's begin with the definition of **COP$_R$** :

$$COP_R = \frac{Q_C}{W_{cycle}} \qquad \textbf{Eqn 3}$$

We can then apply the 1st Law to the cycle :

$$Q_C + W_{cycle} = Q_H \qquad \textbf{Eqn 4}$$

Now, use **Eqn 4** to eliminate **W$_{cycle}$** from **Eqn 3** :

$$COP_R = \frac{Q_C}{Q_H - Q_C} = \frac{1}{\frac{Q_H}{Q_C} - 1} \qquad \textbf{Eqn 5}$$

Now, the Kelvin Principle tells us that, for a reversible cycle like the Carnot Cycle, :

$$\frac{Q_H}{Q_C} = \frac{T_H}{T_C} \qquad \textbf{Eqn 6}$$

Combining **Eqns 5 & 6** gives us :

$$COP_{R,rev} = \frac{1}{\frac{T_H}{T_C} - 1} \qquad \textbf{Eqn 7}$$

We can put numbers into **Eqn 7** : $COP_{R,rev}$ 5

Now, we can calculate the COPR values for the three refrigerators in parts (a) - (c).

Part a.) Use **Eqn 2** to evaluate **COP$_R$** : $COP_{R,A}$ 2.5

Since : $COP_{R,A} < COP_{R,rev}$ we can conclude that this refrigeration cycle is irreversible.

Part b.) Use **Eqn 4** to evaluate **COP$_R$** : $COP_{R,B}$ 5

Since : $COP_{R,B} = COP_{R,rev}$ we can conclude that this refrigeration cycle is **reversible**.

Part c.) Solve **Eqn 2** for **Q$_C$** :

$$Q_C = Q_H - W_{cycle} \qquad \textbf{Eqn 7}$$

Plugging values into **Eqn 7** and then **Eqn 4** gives us :

Q_C	1300	kJ
$COP_{R,C}$	6.5	

Since : $COP_{R,C} > COP_{R,rev}$ we can conclude that this refrigeration cycle is **impossible**.

Part d.) Since : $COP_{R,D} < COP_{R,rev}$ we can conclude that this refrigeration cycle is **irreversible**.

Verify : None of the assumptions made in this problem solution can be verified.

Answers :

a.)	**Irreversible**	c.)	**Impossible**
b.)	**Reversible**	d.)	**Irreversible**

Problem : **WB-3 - A Reversible HE Used to Drive a Reversible Heat Pump - 6 pts**

A reversible power cycle receives $\mathbf{Q_H}$ from a reservoir at $\mathbf{T_H}$ and rejects $\mathbf{Q_C}$ to a reservoir at $\mathbf{T_C}$. The work developed by the power cycle is used to drive a reversible heat pump that removes $\mathbf{Q'_C}$ from a reservoir at $\mathbf{T'_C}$ and rejects $\mathbf{Q'_H}$ to a reservoir at $\mathbf{T'_H}$.

a.) Develop an expression for the ratio $\mathbf{Q'_H / Q_H}$ in terms of the temperatures of the four reservoirs.

b.) What must be the relationship of the temperatures $\mathbf{T_H}$, $\mathbf{T_C}$, $\mathbf{T'_C}$ and $\mathbf{T'_H}$ for $\mathbf{Q'_H / Q_H}$ to exceed a value of **1.0** ?

Read : This problem involves the careful application of the 1st Law to both the **HE** and the **HP**.

The keys is that all of the work produced by the **HE** is used to drive the HP and that the Kelvin Principle applies because both cycles are completely reversible.

After that, the solution is just algebra with the goal of eliminating $\mathbf{Q_C}$ and $\mathbf{Q'_C}$ from the two 1st law Equations.

Given : No numerical values given.
All given information is represented in the diagrams, below.

Find : a.)

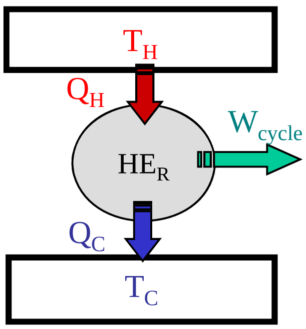
$$\frac{Q'_H}{Q_H} = \text{fxn}\left\{T_H, T_C, T'_H, T'_C\right\}$$

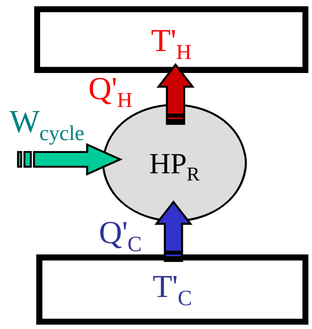
b.) If :

$$\frac{Q'_H}{Q_H} > 1$$

, then what is the relationship between $\mathbf{T_C}$, $\mathbf{T_H}$, $\mathbf{T'_C}$ and $\mathbf{T'_H}$?

Diagram :





Assumptions :

1 - The heat engine operates on a reversible, thermodynamic cycle.
2 - The heat pump operates on a reversible, thermodynamic cycle.

Equations / Data / Solve :

Part a.) Let's begin by applying the 1st Law to both the **HE** and the **HP**.

$$Q_C + W_{cycle} = Q_H \quad \textbf{Eqn 1}$$

$$Q'_C + W_{cycle} = Q'_H \quad \textbf{Eqn 2}$$

Now, solve **Eqns 1 & 2** for $\mathbf{W_{cycle}}$:

$$W_{cycle} = Q_H - Q_C \quad \textbf{Eqn 3}$$

$$W_{cycle} = Q'_H - Q'_C \quad \textbf{Eqn 4}$$

Combining **Eqns 3 & 4** gives us :

$$Q_H - Q_C = Q'_H - Q'_C \quad \textbf{Eqn 5}$$

Now, let's apply the Kelvin Principle to the **HE** and to the **HP**.

$$\frac{Q_H}{Q_C} = \frac{T_H}{T_C} \quad \textbf{Eqn 6}$$

$$\frac{Q'_H}{Q'_C} = \frac{T'_H}{T'_C} \quad \textbf{Eqn 7}$$

Now, we can use **Eqns 6 & 7** to eliminate $\mathbf{Q_C}$ and $\mathbf{Q'_C}$ from **Eqn 5**, as follows.

$$Q_C = Q_H \frac{T_C}{T_H} \qquad \textbf{Eqn 8} \qquad\qquad Q'_C = Q'_H \frac{T'_C}{T'_H} \qquad \textbf{Eqn 9}$$

Plugging **Eqns 8 & 9** into **Eqn 5** gives us:

$$Q_H - Q_H \frac{T_C}{T_H} = Q'_H - Q'_H \frac{T'_C}{T'_H} \qquad \textbf{Eqn 10}$$

Or, after factoring $\mathbf{Q_H}$ and $\mathbf{Q'_H}$:

$$Q_H \left(1 - \frac{T_C}{T_H}\right) = Q'_H \left(1 - \frac{T'_C}{T'_H}\right) \qquad \textbf{Eqn 11}$$

Now, solve **Eqn 11** for $\mathbf{Q_H / Q'_H}$:

$$\frac{Q'_H}{Q_H} = \frac{1 - \dfrac{T_C}{T_H}}{1 - \dfrac{T'_C}{T'_H}} \qquad \textbf{Eqn 12}$$

Or, clearing some of the fractions :

$$\boxed{\frac{Q'_H}{Q_H} = \frac{T'_H \left[T_H - T_C\right]}{T_H \left[T'_H - T'_C\right]}} \qquad \textbf{Eqn 13}$$

Part b.) When : $\dfrac{Q'_H}{Q_H} > 1$ **Eqn 12** becomes :

$$\cancel{1} - \frac{T_C}{T_H} > \cancel{1} - \frac{T'_C}{T'_H} \qquad \textbf{Eqn 14}$$

In **Eqn 14**, the **1**'s cancel, leaving us with :

$$-\frac{T_C}{T_H} > -\frac{T'_C}{T'_H} \qquad \textbf{Eqn 15}$$

Recall that when you multiply an inequality by **(-1)** "greater than" becomes "less than".

So, **Eqn 15** becomes :

$$\frac{T_C}{T_H} < \frac{T'_C}{T'_H} \qquad \textbf{Eqn 16}$$

Or :

$$\boxed{\frac{T_C}{T'_C} < \frac{T_H}{T'_H}} \qquad \textbf{Eqn 17}$$

Verify : None of the assumptions made in this problem solution can be verified.

Answers : a.) $\boxed{\dfrac{Q'_H}{Q_H} = \dfrac{T'_H \left[T_H - T_C\right]}{T_H \left[T'_H - T'_C\right]}}$ b.) $\boxed{\dfrac{T_C}{T'_C} < \dfrac{T_H}{T'_H}}$